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EXPERIMENTAL AND NUMERICAL EVALUATION OF THE INFLUENCE OF STRENGTH MISMATCH ON FATIGUE CRACK GROWTH BEHAVIOR INCLUDING CLOSURE EFFECTS[1]

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Abstract

The integrity of mechanical components, particularly when they experience considerable fatigue damage during its operating life, can be strongly influenced by the presence of residual stress fields and mechanical heterogeneity. Premature closure of crack flanks greatly influences fatigue crack growth rate. Extensive elastic-plastic finite element analyses have been carried out to investigate detailed crack closure behavior in center cracked welded compact tension (CT) specimens with one level of weld strength mismatch. The finite element results show that homogeneous, soft material has higher crack opening loads than heterogeneous material with 50% overmatch conditions. Fracture testing conducted on C(T) specimens to measure fatigue crack growth rates for an ASTM A516 Gr. 70 steel weldment provide the experimental data to support such behavior. The fatigue life can be reduced by more than 100 % for a condition of 50% overmatch when compared with the evenmatch condition.

Keywords: Mismatch; Fatigue life; Crack closure.

AVALIAÇÃO EXPERIMENTAL E NUMÉRICA DA INFLUÊNCIA DA HETEROGENEIDADE MECÂNICA SOBRE O COMPORTAMENTO DE CRESCIMENTO DE TRINCA POR FADIGA INCLUINDO EFEITOS DE FECHAMENTO

Resumo

A integridade de componentes mecânicos, especialmente quando sofrem considerável dano sob fadiga durante sua vida operacional, pode ser fortemente influenciado pela presença de campos de tensões residuais e heterogeneidade mecânica. O fechamento prematuro dos flancos da trinca influencia a taxa de crescimento da trinca. Análises elasto-plásticas usando elementos finitos (EF) foram realizadas para investigar o comportamento detalhado do fechamento de trinca no centro do cordão de solda de corpos compacto (CT) com um nível de heterogeneidade mecânica (Mismatch) da junta soldada. Os resultados de EF mostram que o material homogêneo tem cargas de abertura maiores da que o material heterogêneo com condições overmatch 50%. Ensaios realizados nos corpos C (T) soldados para medir as taxas de crescimento de trinca por fadiga de um aço ASTM A516 Gr.70 fornecem os dados experimentais para suportar o comportamento obtido por EF. A vida à fadiga pode ser reduzida em mais de 100% para uma condição de overmatch 50% quando comparada com a condição evenmatch.

Palavras-chave: Heterogeneidade mecânica; Vida à fadiga; Fechamento de trinca.

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1 INTRODUCTION

Fatigue assessments of steel weldments remain a key issue in design, fabrication and safe operation of critical engineering structures including pressure vessels, storage tanks and piping systems among others. Typical welding processes (e.g., hot or cold cracking, lack of penetration, undercut) almost invariably introduce crack-like defects in the welded region, most often in the form of planar flaws, which substantially reduce the fatigue strength of the welded joint.

Current design and in-service fatigue assessments for welded components in pressure vessels and piping systems, including the provisions given in the ASME(1) code, employ conventional procedures adopting stress-life approaches in which fatigue life predictions are made based upon an implicit crack initiation criterion and the simple knowledge of nominal stresses acting on the weldment. While used effectively in many structural applications, it is now generally recognized that crack propagation from pre-existing defects dominates the fatigue life of welded joints. Moreover, there is a complex interplay of key factors affecting fatigue crack growth rates in welds, such as weld geometry, strength mismatch and weld residual stresses, which is not explicitly taken into account in conventional stress-life methodologies. Consequently, advanced procedures for fatigue assessments of critical weldments must include the effects of crack extension under cyclic loading on fatigue life predictions for structural components. Moreover, accurate estimates of the effective crack-tip driving force (here characterized by the stress intensity factor (SIF) range, as measured by ΔK) with crack extension become central in robust correlations of fatigue behavior between these conventional test specimens and structures.

As the crack advances under cyclic loading, material flow takes place in the plastic zone surrounding the crack tip. This highly deformed material remains adjacent to the crack faces causing closure of the crack tip above the minimum applied load. The residual near-tip strains associated with the cyclic loading (tensile strains during loading are not fully reversed at the unloading) form a plastic wake, which is responsible for the premature closure of the crack. Such mechanism may strongly affect the effective load increment and reduce the Fatigue Crack Growth Rate (CGR).

This work provides an investigation of the influence of strength mismatch on FCGR behavior for an ASTM A516 Gr. 70 steel weldments. Recent finite element analyses conducted by Sarzosa and Ruggieri(2) revealed that strength mismatch (overmatch) has a detrimental effect on the fatigue crack propagation life. Thus, one purpose of this study is to develop a framework for evaluation of crack closure effects on FCGR for steel weldments while, at the same time, gaining additional understanding of key factors affecting the fatigue life of structural welds. Another intention is to assess the potential strong effects of plasticity-induced crack closure (PICC) on fatigue crack extension of weldments. Very detailed non-linear finite element analyses using plane-strain models of compact tension (CT) fracture specimens with center cracked, square groove welds provide the evolution of crack growth with cyclic stress intensity factor which is required for the estimation of the closure forces. Fatigue life estimation using different approaches provides the more “appropriate” numerical/experimental methodology to take in account closure effects for mismatched weldments.

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2 MATERIAL AND EXPERIMENTAL PROCEDURE

The material tested was an ASTM A516 Grade 70 steel plate of 16 mm thickness. The welded joint was made by multiple manual pass shielding metal arc welding process (SMAW), using electrode FOX-7018, with diameter 3 mm. The chemical composition and mechanical properties are shown in Table 1 and Table 2, respectively. The mechanical properties reported correspond to the average of 3 tensile specimens for the base and weld metal.

Table 1. Chemical composition of A516 steel (% Weight)

C	Mn	P	S	Si	Al	Cu	Cr	Ni
0.22	1.04	0.01	0.008	0.209	0.033	0.014	0.02	0.011

A low toughness orientation, where the crack propagates in the rolling direction was adopted (T-L orientation). Specimen geometry and experimental procedure were compatible with the requirements of ASTM E 647-08.[3] A compact tension (CT) specimen was used to perform the experimental test. A sketch of the specimen is shown in Figure 1. A clip gage was located in a V-notch machined at the mouth of the CT specimen to measure the crack length.

Table 2. Monotonic tensile test properties of A516 steel and its weld metal

Specimen	$\sigma_{Yield}\,(\mathrm{MPa})$	$\sigma_{Ultimate}\,(\mathrm{MPa})$
Base metal	343	554
Weld metal	510	580

Fatigue crack growth (FCG) experiments were conducted on a servo-hydraulic MTS-810 testing machine at a frequency of 30 Hz, sine wave, in air at room temperature. All tests were conducted under load control, constant amplitude loading, and load ratio R=0.1 The MTS software calculated the crack length, based on CMOD gage displacements signal, using a compliance equation. For each crack increment of $\Delta a = 0.1\,\mathrm{mm}$ the software store the crack length, cycle number, maximum load, minimum load. The specimens were precracked under constant load ratio R=0.1, 30 Hz, and decreasing maximum load until reached an initial crack length (notch + precrack) of $a = 11.2\,\mathrm{mm}$. The FCG test started with $K_{max} = 12\,\mathrm{MPa}\sqrt{\mathrm{m}}$, which corresponds to applied remote load equal to $P_{max} = 7067.3\,\mathrm{N}$.

All specimens were stress relieved, after machined to the final size, by heat-treated. A post-weld head treatment at $590^{\circ}C$ for 30 minutes, with controlling rate of heating equal to $3^{\circ}\frac{C}{\min}$ from $425^{\circ}C$ and rate of cooling equal to $5^{\circ}\frac{C}{\min}$ in average until $450^{\circ}C$

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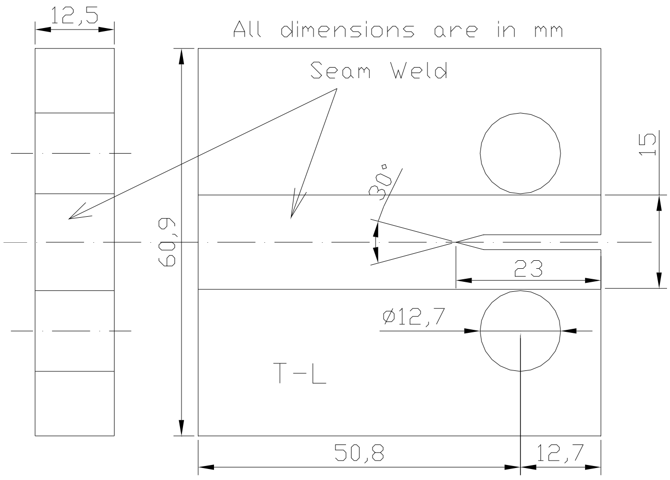

Figure 1. Compact tensile specimen used in the experimental test of fatigue crack propagation.

2.1 Crack Opening Load Evaluation

The ASTM E647-08[3] standard approach defines the opening load, P_{op}, analyzing the compliance variations during the loading part of the load cycle with respects to the compliance of the unloading portion of the load cycle. This determination of P_{op} is based on the observation that when a cracked specimen is loaded up to the force at which the crack becomes fully open, the compliance attains a characteristic value and remains constant upon further loading increase until the force is increased enough to cause large-scale yielding at the crack tip. Upon unloading, the compliance has the characteristic value for the fully open crack regardless of the level of plasticity reached during the loading. Therefore, conceptually the task consists in determine the force at which the displacement (strain) against force trace become linear during loading. However, fully open load results in only a subtle change in compliance that is difficult to identify in experimental data. Thus, the definition of compliance offset was introduced to reduce scatter in opening force results due to noise and non-linearity. P_{op} is defined as the force corresponding to a compliance that is offset from the fully open crack value. The MTS software has a fitting approach (quadratic + linear) and offset compliance approach to determine the opening load for each crack length.

Alternately, the finite element method appears as a powerful numerical tool for estimating crack tip shielding. P_{op} has been defined typically as the applied remote load at which the first node behind the crack tip disjointed from the crack plane. Adequate meshing, around the crack tip, must be performed to simulate correctly the reversed plastic zone and guarantee that the results are not mesh dependent. Sehitoglu and Sun[4] have proposed different criterion. They stated that the crack opens when the stress state ahead of crack tip, which was compressive in the previous cycle, changes from compression state to tension. Consequently, they measured the opening load when the crack tip nodal stress, normal to crack plane, changes from negative to positive value. Such criteria appears to be more realistic than displacement criteria because the node behind the advancing crack tip can experience a positive displacement, but the crack tip node would still carry compressive stresses.

Recently, Roychowdhury and Dodds[5] proposed to monitor the second node behind the crack tip to minimize any influence of the finite element approximation near the crack tip. The consistency and reliability of these three numerical criteria will be tested here to calculate P_{op} and fatigue life, N_f, for two fracture specimens: welded and non-welded. These N_f estimations will tell us the accuracy of numerical approaches when compared with experimental data.

2.2 Fatigue Crack Growth Modeling

Evaluation of the safe service life for structures containing cracks under cyclic loading has been usually conducted based on semi-empirical relationships connecting key parameters characterized by the crack growth rate da/dN, stress intensity factor range ΔK and R-ratio, in addition to some material properties whose values are commonly determined by experimental measurements. The standard approach to predict fatigue crack growth adopts the Paris law to assess the crack growth rate per cycle of load da/dN.[(6)] The rate of crack growth is assumed to be a function of the effective stress intensity factor (ΔK_{eff}), which is defined as the difference between the maximum stress intensity factor, K_{max}, and the opening stress intensity factor, K_{op}. The basic algorithm involves building a refined enough mesh for the loads to be applied; it means enough elements on the crack tip in order to simulate the effects of forward and reverse plastic zone. Remote loads are applied cyclically (time-dependent) and the crack tip is released at an arbitrary point on the cycle, either during the loading or unloading portion. This is repeated for each applied cycle. This approach does not consider the physics of the real problem and only intends to capture the history of the development of plastic wake. Here, we are interested in quantifying the PICC phenomena.

The growing crack creates a plastic wake that is responsible for the premature contact of the faces of the crack, even for loads greater than zero. The process is repeated until stable values of K_{op} are obtained.

In this study a constant cyclic loading was imposed on the model under load control conditions. The cyclic loading consisted of repeatedly increasing the values of applied load from minimum load (P_{min}) to maximum load (P_{max}) and then decreasing back to P_{min} as shown in Figure 2. The crack propagates uniformly by an amount equal to Δa =0.02mm in each cycle. This is achieved by releasing all (current) crack front nodes after the maximum load is applied. Here the amount of crack growth was set equal to two times the size of plastic zone, $2r_p$, for plane strain condition calculated as a second order estimation using Irwin's approach.[(7)]

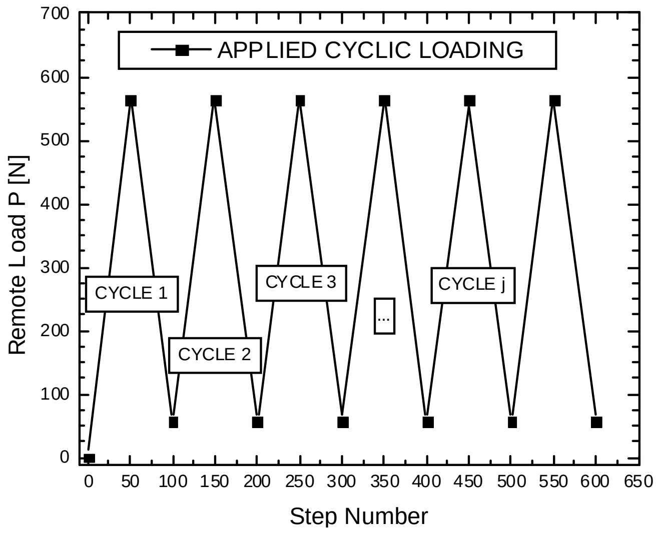

Figure 2. Constant incremental load applied by each cycle.

Detailed finite element analyses were performed on plane-strain models for conventional 1-T compact tension, C(T), fracture specimens. The research code WARP3D was employed[7] in the numerical computations reported here. Symmetry conditions permit modeling of only one-half of the specimen with appropriate constraints imposed on the remaining ligament. Thus, the number of nodal degree of freedom is reduced and consequently the computational time is decreased.

A typical half-symmetric model has 2059 8-node, 3-D elements (4472 nodes) as shown in Figure 3. One layer was used to define the thickness of the specimens. A sequence of small and identical elements ahead of crack front permits an equal amount of crack growth, as shown in Figure 3. The material properties for the elements around the loading point are defined as an elastic material to enhance numerical converge. The analyses use an incremental, isotropic hardening constitutive model to describe the cyclic, elastic-plastic response of the material in small geometry change setting. The numerical solutions employ a simple linear, power-hardening model to characterize the uniaxial true stress-logarithmic strain in the form:

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\bar{\sigma}}{\sigma_y} \quad \varepsilon \leq \varepsilon_y \quad ; \quad \frac{\varepsilon}{\varepsilon_y} = \left(\frac{\bar{\sigma}}{\sigma_y}\right)^n \quad \varepsilon > \varepsilon_y \qquad (1)$$

where σ_y and ε_y are the yield stress and strain, and n is the strain hardening exponent. The finite element analyses consider material flow properties for the ASTM A516 Grade 70 (20º C) reported in Table 2. The strength mismatch level M_y is defined as:

$$M_y = \frac{\sigma_y^{WM}}{\sigma_y^{BM}} \qquad (2)$$

where σ_y is the yield stress, and superscripts WM and BM denoting weld metal and base metal.

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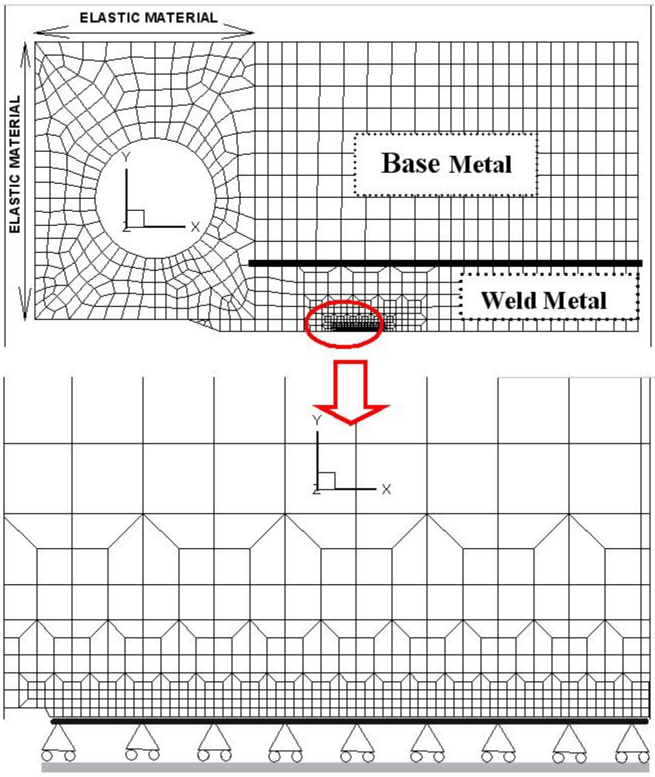


Figure 3. Details of the mesh used in the analysis.The crack tip area has been enlarged to ilustrate the mesh around the crack tip.

The adopted yield stress and hardening properties for the base plate and weld material, utilized in the numerical simulations of these fracture specimens with square groove welds, which also consider E = 206 GPa and $\nu = 0.3$, are shown in Table 3.

Table 3: Material properties adopted in the analyses

MISMATCH	WELD		BASE PLATE	
	$\sigma_y\,(\mathrm{MPa})$	n	$\sigma_y\,(\mathrm{MPa})$	n
50% Overmatch	510	17.93	343	6.85
Evenmatch	343	6.85	343	6.85

The parameter $\mathrm{P_{op}}$ was determined from 4 different approaches including numerical strategies: (a) Node displacement of first node behind crack tip $\mathrm{u_y^1}$; (b) Node displacement of second node behind crack tip $\left(\mathrm{u_y^2}\right)$; (c) Crack tip stress $\left(\sigma_{yy}^{tip}\right)$; and experimental procedure represented by curve fitting of load-displacement data.

3 RESULTS AND DISCUSSION

3.1 Residual Stresses

The residual stresses on the specimens were determined using the technique of X-Ray diffraction. X-ray diffraction uses the distance between crystallographic planes as strain gage. The deformation cause changes in the spacing of lattice plane from their stress free value to a new value that corresponds to the magnitude of the residual stress. A detailed explanation of this technique can be found in Anderoglu.(8)

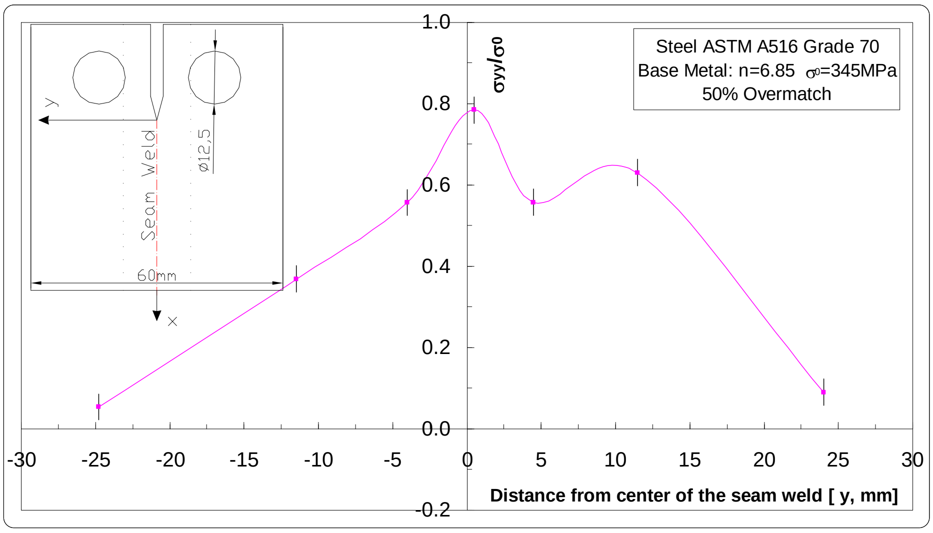
The diffraction measurements were carried out on a SHIMADZU XRD-700 diffractometer. The average of four incidence angles ($0^{\circ}, 15^{\circ}, 30^{\circ}$, and 40°) was used to calculate the residual stresses. The distribution of residual stresses normal to crack plane (σ_{yy}), normalized by σ_{y}^{BM}, with the distance from the center of seam weld is shown in Figure 4.

We can see that heat treatment almost relieves the residual stresses completely. The residual stresses will be an important factor to be considered when evaluating fatigue life, especially for low applied fatigue loads (reduced plastic deformation at the crack tip). This remnant residual stress can be used as a one factor to explain the reduced fatigue life for the welded specimen analyzed later.

3.2 Experimental Results of FCG Test

Figure 5 shows the experimental evolution of crack length with the number of cycles of the applied load for both specimens (welded and homogeneous). The welded specimen had a lower “performance” to bear fatigue loads than the homogeneous one. This difference can be explained, at least partially, using the crack closure concept. Considering PICC as the primary source of closure at the crack tip. The higher yield stress for the welded specimen “constraint’ plastic deformation at the crack tip, so there is no formation of a significant wake behind the crack tip and little closure happens for this specimen. In contrast, the homogeneous specimen, with lower yield stress, develops considerable plastic deformation at the crack tip and a plastic wake is created, resulting in crack tip shielding from remote load. Thus, an effective methodology that estimates this shielding is fundamental in fatigue life assessment of structural components for planning periodic-safety inspections.

Another key factor that may explain the higher propagation rate for welded specimen is the remaining residual tensile stresses that were not relieved in the specimen after heat treatment (Figure 4). It is well known that residual tensile stresses increase the mean stress and reduce the weldments fatigue strength. Although the magnitude of remnant tensile stress is low, the crack tip will experience a stress ratio higher than the applied R, increasing the crack growth rate. Only the experimental data shown in Figure 5 is reliable. Any further calculations or estimations will be based on assumptions or simplifying hypothesis, for example the Paris law used to calculate N_f. Our goal here will be to calculate the driving force ΔK_{eff} at the crack tip and use the Paris law to estimate the fatigue life.



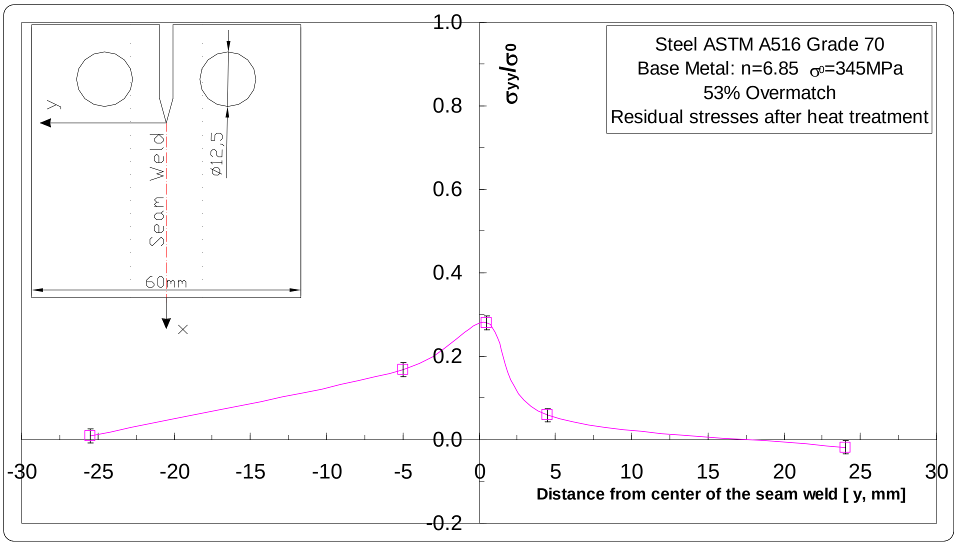


Figure 4. Distribution of residual stress σ_{yy} in the welded specimen (a)) before heat tratment, and (b) after heat treatment.

The fatigue crack growth rate $\left(\frac{da}{dN}\right)$ was estimated using the secant method. The integration of this "linear" relationship between $\frac{da}{dN}$ and ΔK will help us to make estimations about the fatigue life propagation for the welded and homogeneous specimen. Since ΔK increases with crack length during constant amplitude loading, the growth rate is not constant so it is necessary to use an integration procedure to obtain the life required for crack growth. The procedure employed to perform the integration of FCGR was taken from Dowling.[6] Basically, the life in cycles is determined by the integration of the inverse of FCGR from an initial size ai to a final size af $\left[N_f = \int_{a_i}^{a_f} \left(\frac{dN}{da}\right) da\right]$. The inverse of FCGR, $\frac{dN}{da}$, represents the rate of accumulation of cycles per unit increase in crack length.

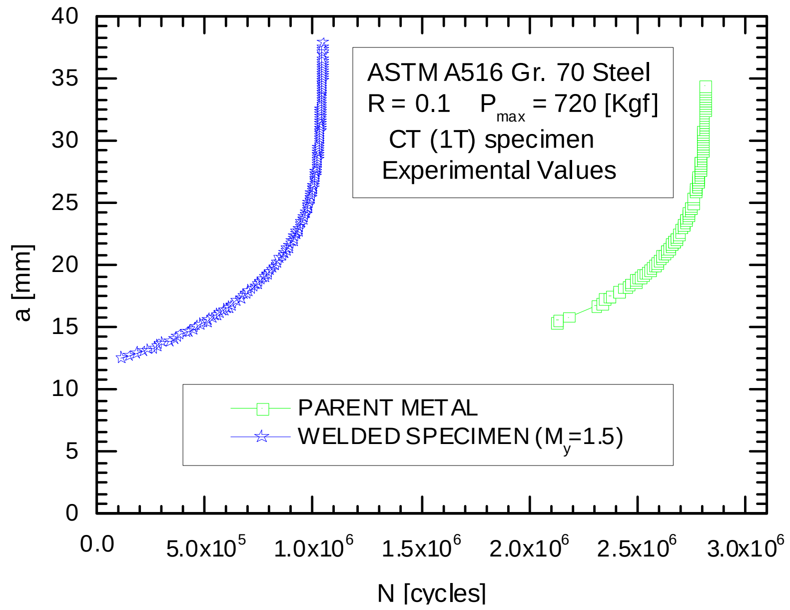


Figure 5: Crack length versus the number of cycles.

The numerical results are compared with the analytical solution presented by Newman,[9] which employed the strip yield model to calculate P_{op}. Newman's equation for calculation of opening loads has been employed extensively by industry and academia. Further, Newman's equation is implemented in FASTRAN software principal numerical code used to predict fatigue life for constant and variable cyclic load, so that the employed methodology can be also evaluated with direct comparison with this theoretical solution.

3.3 Numerical Results

The opening load was evaluated for a range of K_{max} and the results are shown in

Figure **6**. The experimental measures of P_{op} and analytical calculation by Newman's model are also included as reference. In the evenmatch condition, the difference between σ_{yy}^{tip} and u_y^1 is almost 11% for low loads and 20% for high loads. For 50% Overmatch, a bigger difference (20%) is found for low loads and similar 20% for high loads. The u_y^2 approach detects small levels of closure. Comparing the response of both specimens, it can be stated that greater levels of plastic deformation at the crack tip needs greater values of the applied remote force to open crack flanks. The lower σ_y in the homogeneous specimen gives rise to higher P_{op} values when compared to welded specimen.

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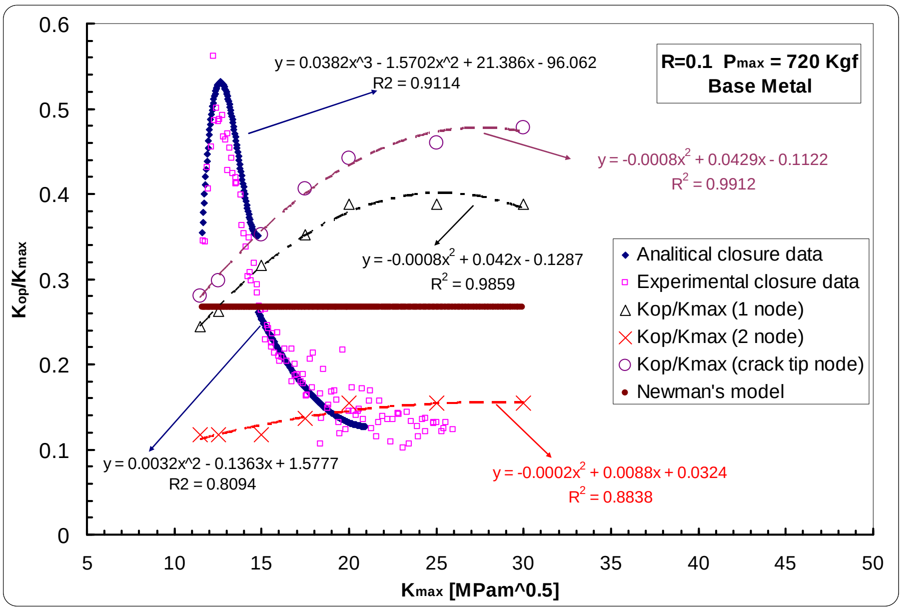

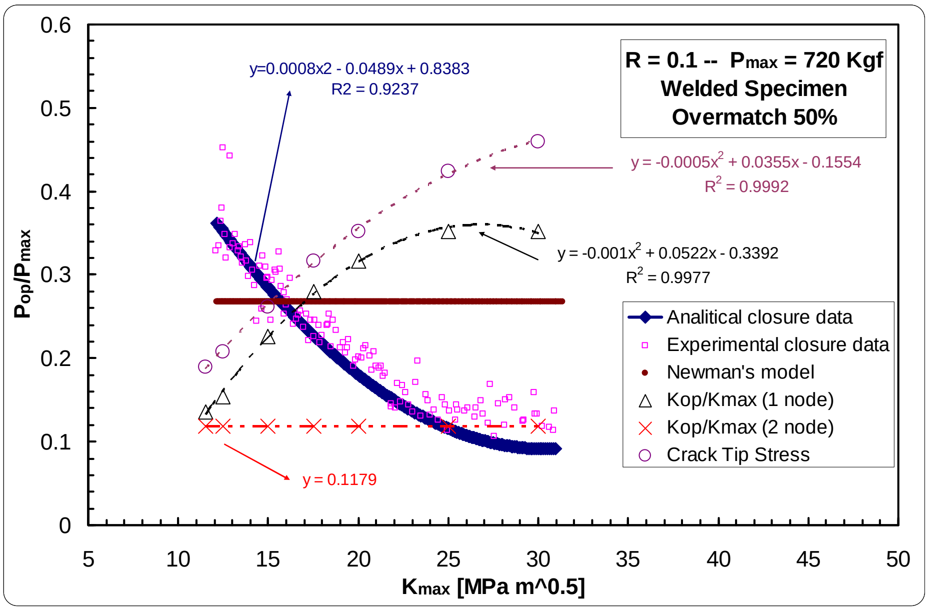

Figure 6: Values of P_{op}/P_{max} as function of maximum applied stress intensity factor for R = 0.1, and P_{max} = 563 N.

The numerical P_{op} values, obtained by σ_{yy}^{tip} and u_y^1, increase with the level of the applied load until the constraint to plastic deformation is lost. Then, P_{op} decreases as shown in

Figure **6**. This tendency has been reported experimentally by Asbaugh[10] and Guo.[11] The experimental results of P_{op} show a similar tendency, except at the beginning of the test for the homogeneous specimen where first a sudden increment of P_{op} was measure. P_{op} values for homogeneous specimen were appreciable higher than welded. Newman's model results in nearly constant opening values.

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Least-square fitting has been performed for each data and the obtained regression used to calculate ΔK_{eff} with the increment of crack size.

3.4 Fatigue Life Predictions (FLP)

The FCGR was expressed as a function of ΔK_{eff} as proposed by Elber[12]. The integration of FCGR-ΔK_{eff} relationship, $\frac{da}{dN}=C\left(\Delta K_{eff}\right)^{m}$, to obtain N_f, was performed by trapezoidal rule using increments of crack length of 0.05mm. The initial crack size was 11.25 mm and the final crack size was 36.37 mm and 37.87 mm for homogeneous and welded specimen, respectively. The crack growth estimations are shown in Figure 7. Besides, the curve of fatigue life without taking in consideration the closure effect has been included as reference.

The variation between the FLP showed in Figure 7 lies in the different values of ΔK_{eff} estimated for each model. The finite element results of P_{op} were fitting by a quadratic function. As the fatigue life is mostly consumed during the initial crack growth the values of P_{op} that has more impact in fatigue estimations corresponds to "low" K_{max}. As an example, the crack spent 94% and 84% of its fatigue life in propagate the initial 10 mm for the homogeneous and welded specimen, respectively, as shown in Figure 5.

Better approximations of N_f are obtained with the u_y^1 approach and Newman's model for homogeneous specimen. Each one has an absolute error of 14% when compared to the experimental data. However, different approach, u_y^2, got better results for welded specimen. For this specimen, the u_y^1 method overestimates N_f by 60% and u_y^2 predicts exactly the same life than experimental data.

4 CONCLUSIONS

Experimental testing and finite element simulation under constant amplitude loading with R=0.1 were performed for homogeneous and welded specimens ($M_y=1.5$) of a compact tension specimen. Different approaches were used to calculate the effective crack driving force. The differences in fatigue crack propagation between homogeneous and welded specimen can be explained using the crack closure concept. The conclusions drawn for this study are as follows:

- The results indicate that heterogeneity in mechanical properties between the base plate and weld metal has a great effect on crack opening behavior. The homogeneous, soft, material has larger crack opening loads than welded 50% overmatch specimen. This results in longer fatigue life for homogeneous specimen than the welded due to reduce ΔK_{eff}.

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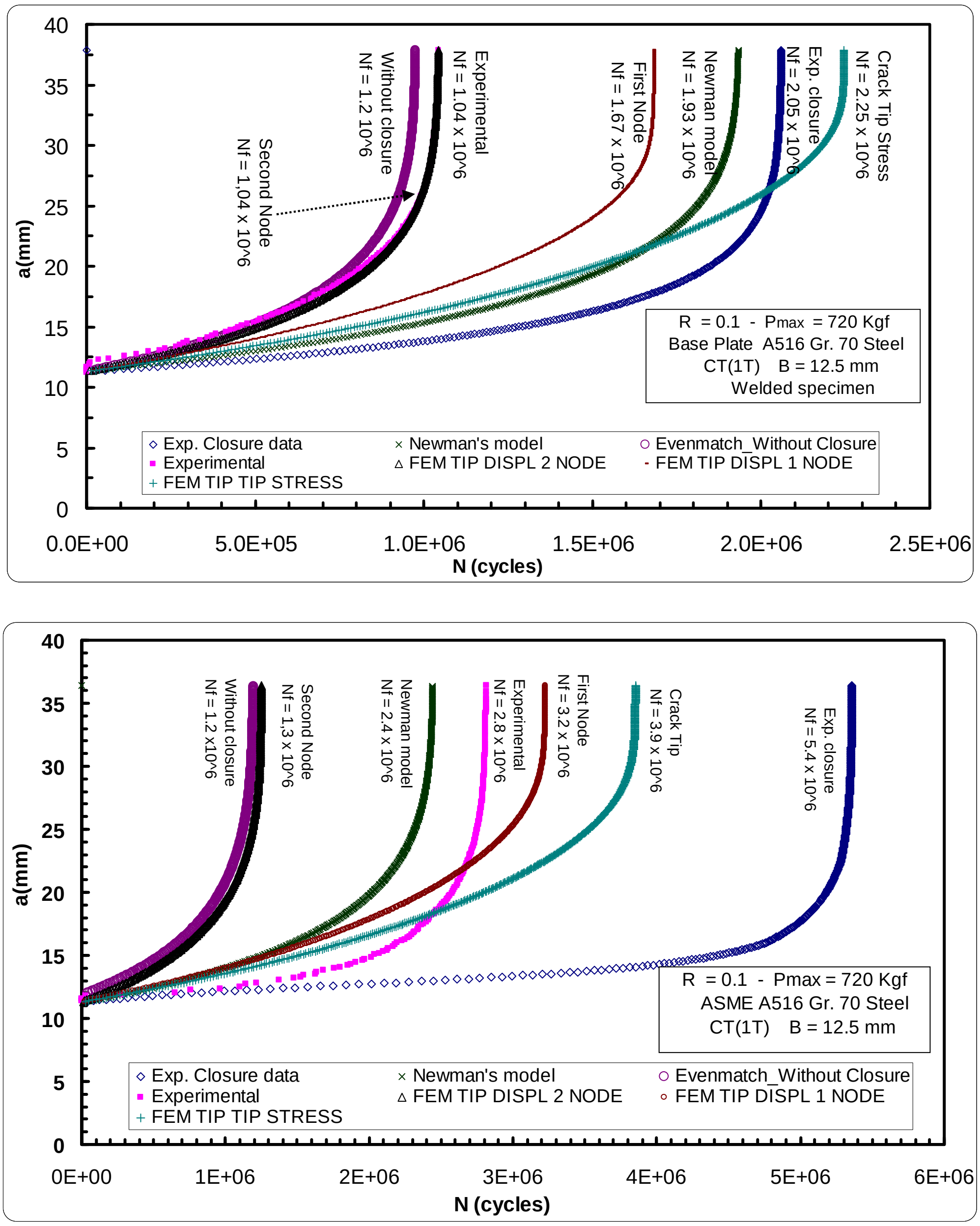

Figure 7. Comparisons of fatigue life propagation, for constant amplitude loading, using different approaches to get ΔK_{eff} .

- The crack tip shielding during cycling loading is associated with the plastic deformations left behind the crack tip with the advance of crack. Overmatch weldments with higher yielding stress “constraint” plastic deformation at the crack tip and consequently decreasing the shielding effect due to closure of crack.
- The approach that uses the displacement of the first node behind the crack tip yields better estimations of fatigue life than the second node displacement and tip stress for homogeneous specimen. Displacement of second node behind the crack tip gave excellent agreement to estimate fatigue propagation life.
- Overmatch conditions in welded joints have detrimental effects on the fatigue crack propagation life.

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- The experimental closure load obtained by curve fitting, quadratic + linear, of load-CMOD data reported by MTS software was no able to correlate experimental crack growth. Thus, the curve fitting measurements approach produces high closure levels so the estimated fatigue life is overestimated.

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