FATIGUE CRACK PROPAGATION SIMULATION IN PLANE STRESS CONSTRAINT[1]

Luiz Carlos H. Ricardo [2]
Dirceu Spinelli [3]

Abstract

Nowadays, structural and materials engineers develop structures and materials properties using finite element method. This work presents a numerical determination of fatigue crack opening and closure stress intensity factors of a C(T) specimen. Two different standard variable spectrum loadings are utilized, Mini-Falstaff and Wisper. The effects in two-dimensional (2D) small scale yielding models of fatigue crack growth were studied considering plane stress constraint.

Keywords: Fatigue; Crack propagation; Simulation; Small scale yielding model.

SIMULAÇÃO DA PROPAGACÃO DE TRINCAS POR FADIGA NO ESTADO PLANO DE TENSÃO

Resumo

Hoje em dia, engenheiros estruturais e de materiais desenvolvem componentes e propriedades dos materiais utilizando o método dos elementos finitos. Este trabalho apresenta um método numérico para a determinação dos fatores de intensidade de tensão de abertura e fechamento de trincas de fadiga em corpo de prova padrão ASTM C(T). Dois diferentes espectros de carregamentos padrões de amplitude variável são utilizados, Mini-Falstaff e Wisper. Os efeitos em modelos bidimensionais (2D) de crescimento de trincas em escoamento de pequena escala são estudados considerando estado plano de tensão.

Palavras-chave: Fadiga; Propagação de trincas; Simulação; Escoamento em pequena escala.

---

[1] *Contribuição técnica ao 65º Congresso Anual da ABM, 26 a 30 de julho de 2010, Rio de Janeiro, RJ, Brasil.*

[2] *Assistant Professor, Aalborg University, Denmark*

[3] *Full Professor, University of Sao Paulo , Brazil*

1 INTRODUCTION

The most common technique for predicting the fatigue life of automotive, aircraft and wind turbine structures is Miner's rule.(1) Despite the known deviations, inaccuracies and proven conservatism of Miner's cumulative damage law, at present it is being used in design of many advanced structures. Fracture mechanics techniques for fatigue life predictions remain as a back-up in design procedures. The most important and difficult problem in using fracture mechanics concepts in designs seems to be the use of crack growth data in predicting fatigue life. This experimentally obtained data is used to derive a relationship between stress intensity range (ΔK) and crack growth per cycle (da/dN).

In cases of fatigue loaded parts containing a flaw under constant stress amplitude, crack growth can be calculated by simply integrating the relationship between *da/dN* and ΔK. However, for complex spectrum loading, simple addition of the crack growth occurring in each portion of the loading sequence produces results that very often are more erroneous than those obtained using Miner's rule in an *S-N* curve. Retardation tends to cause conservative Miner's rule life predictions where the fatigue life is dominated by crack growth. However, the opposite effect generally occurs where the life is dominated by the initiation and growth of small cracks. In these cases, large cyclic strains, which might occur locally at stress raisers due to overload, may pre-damage the material and lower its resistance to fatigue. This effect is generally handled by basing the crack initiation life prediction on a modified (lowered) strain-life or stress-life curve that includes the effect.

In 1960 Schijve(2) observed that experimentally derived crack growth equations were independent of the loading sequence and depended only on the stress intensity range and number of cycles for a given portion of loading sequence. The central problem in the successful utilization of fracture mechanics techniques applied in a fatigue spectrum is to obtain a clear understanding of the influence of loading sequences on fatigue crack growth. Of particular interest in the study of crack growth under variable amplitude loading is the decrease in the growth rate called crack growth retardation that usually follows a high overload. Most of the reported theoretical descriptions of retardation are based on data fitting techniques, which tend to hide the behavior of the phenomenon. If the retarding effect of a peak overload on the crack growth is neglected, the prediction of the material lifetime is usually very conservative.(3) The small scale yield model employs the Dugdale(4) theory of crack tip plasticity, modified to leave a wedge of plastically stretched material on fatigue crack surfaces. Fatigue crack growth was simulated by Skorupa and Skorupa(5) using the strip model over a distance corresponding to the fatigue crack growth increment as shown in Figure 1.Falstaff and Wisper are generated by Genesis(6) that is a fatigue code used to generate the standards spectrum loadings for some of the mentioned application like Falstaff for aeronautics and Wisper for wind turbine components.

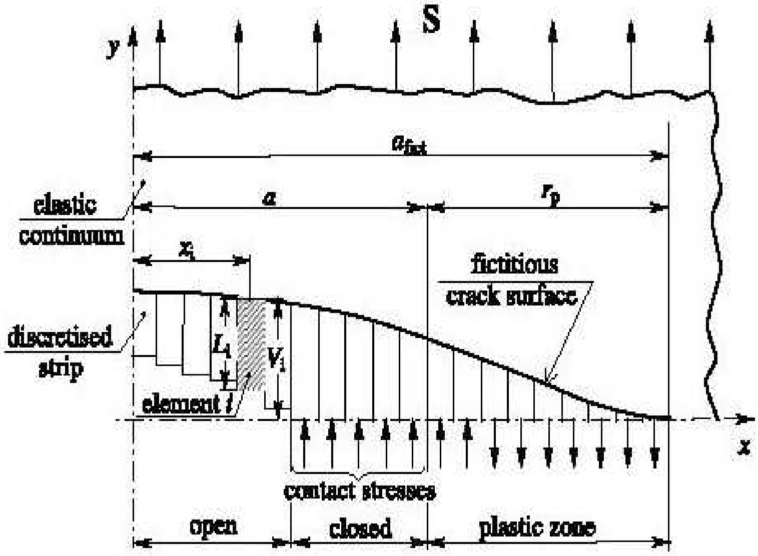

Figure 1. Schematic small scale yield model(5)

In order to satisfy the compatibility between the elastic plate and the plastically deformed strip material, tensile stress must be applied on the fictitious crack surfaces. Tensile stresses are also needed over some distance ahead of the crack tip in the crack wake region as shown in Figure 1 ($a_{open} \leq x < a$), where a_{open} indicates the crack opening, when the plastic elongations of the strip *L(x)* exceed the fictitious crack opening displacements, *V(x)*, and in the plastic zone ($a \leq x < a_{fict}$), where a_{fict} indicates a fictitious crack extension as in the original Dugdale model.

In the real world of application like automotive, aeronautic, naval and wind turbine for example, the loading history are random and it is necessary edit the signal in a way that the edition does not affect the quality of results when used for numerical and experimental activities. In the literature it is unlikely to find works regarding the procedure to determine when crack opening or closing due to random loading cycle by cycle be normally used by blocks of cycles with the same amplitude.

2 LITERATURE REVIEW

Newman(7) develops the first crack advance scheme model with a simulation covering crack closure mechanism using finite element method having the experimental work of Elber(8) as a baseline. To determine the crack closure stress a quarter of CCT specimen was used and the changing boundary conditions associated with intermittent opening and closing of the crack surfaces were accommodated through a series of truss elements along the crack line. The stiffness of a given truss was set to an extremely large value when the crack was “closed” at that location and set to a negligibly small value when the crack was “open.” Stresses and displacements along the crack line were monitored on each load increment to determine if boundary conditions should be changed at any location. Blom and Holm(9) worked with a similar procedure developed by Newman.(7)

The crack propagation was simulated by releasing the crack tip node during each cycle at the maximum applied stress and then changing the boundary conditions. Newman(10) presented a modified procedure was presented based on Newman,(7) but with a crack tip strain criterion for crack extension.

A literature review on plasticity-induced fatigue crack closure under plane strain conditions phenomenon is presented in Table 1.(11) This article also mentions that there are controversial topics concerning the mechanics of crack propagation. Matos & Norwell(11) present at Table 1 a literature review of the phenomenon of plasticity-induced fatigue crack closure under plane strain conditions and mention that there are controversial topics concerning the mechanics of crack propagation. In general, there is no consensus in the scientific community. Sander and Richard(12) cover the effects of overload using, FELIX/28(13) includes the spectrum loading in a compact tension specimen and compare to numerical and experimental data.

Table 1. Chronological crack advance scheme

Year	Author	Node Release Scheme	Constraint	Target	Element Type
1974	Newman(7)	Maximum load	PStress	COP and CCL	Triangle linear
1985	Blom and Holm(9)	Maximum load	PStress; PStrain	COP and CCL	Triangle linear
1989	McClung and Sehitoglu(14)	Maximum load	PStress; PStrain	COP	Quadrilateral linear
1990	McClung et al.(15)	Maximum load	PStress; PStrain	COP	Quadrilateral linear
1991	Sehitoglu and Sun(16)	Maximum load	PStress; PStrain	COP	Quadrilateral linear
1992	Sehitoglu and Sun(17)	Maximum load; Minimum load	PStress; PStrain	COP	Quadrilateral linear
1996	Wu and Ellyin(18)	Maximum load	PStress	COP and CCL	Quadrilateral linear
1999	Ellyin and Wu(19)	Maximum load	PStress	COP and CCL	Quadrilateral linear
2000	Wei and James(20)	Maximum load	PStress; PStrain	COP and CCL	Triangle linear
2002	Ricardo et al.(21)	Minimum Load	PStress	COP and CCL	Triangle quadratic
2003	Ricardo(22)	Minimum Load	PStress	CCL	Triangle quadratic
2003	Solanski(23)	Maximum load	PStress; PStrain	COP and CCL by COEL	Quadrilateral linear

PStress- plane stress; PStrain- plane strain; COP- crack opening; CCL- crack closing; COEL- crack opening and closing by contact element; CME- crack opening and closing by compliance method; DME- crack opening and closure by displacement method.

3 RETARDATION PHENOMENON

Corbly & Packman(24) describe some aspects of the retardation phenomenon. Despite the recent increase in research of retardation effects in crack propagation, there are many aspects of load interaction phenomena that lack adequate explanations. Retardation aspects that are generally agreed upon are presented below.

- Retardation increases for higher values of peak loading, $\sigma_{peak,}$ for constant values of lower stress levels.(25,26)
- The number of cycles at the lower stress level required to return to the non-retarded crack growth rate is a function of ΔK_{peak}, ΔK_{lower}, R_{peak}, R_{lower}, and number of peak cycles.(27)

- If the ratio of the peak stress to lower stress intensity factors is greater than about 1.5, then complete retardation (arrest) at the lower stress intensity range is observed. However, some tests may not have continued long enough to verify if the crack ever propagated again.(27)
- With a constant ratio of peak to lower stress intensity, the number of cycles to return to non-retarded growth rates increases with increasing peak stress intensity.(26,27)
- Given a ratio of peak stress to secondary stress, the number of cycles required to return to non-retarded growth rates decreases with increased time at zero loading before cycling at the lower level.(27)

4 DESCRIPTION OF MODEL

A compact tension specimen was modeled using a commercial finite element code, MSC/Patran, r1(28) and ABAQUS Version 68(29) used as solver. Half of the specimen was modeled and symmetry conditions applied. Figure 2 shows the compact tension specimen from ASTM 647-E95a model used in the present work. A plane stress constraint is modeled by the finite element method covering the effects in two dimensional (2D) small scale yielding models of fatigue crack growth variable spectrum loading. The boundary conditions are presented in Figure 3. The finite element model has quadrilateral elements, S8R, with quadratic formulation and spring elements, SPRING1.

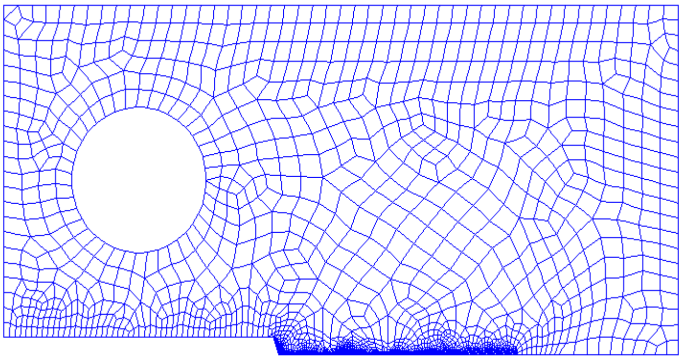

Figure 2. Half compact tension modeled by finite element method.

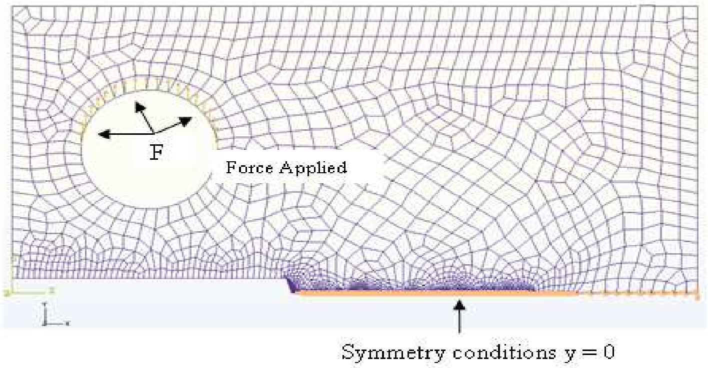

Figure 3. Boundary conditions used in the FEM Crack propagation model

FALSTAFF (**F**ighter **A**ircraft **L**oading **STA**ndard **F**or **F**atigue evaluation) is a loading sequence standard representative of the load history of the wing root of a fighter aircraft. Mini-FALSTAFF is a reduced load history that maintains the properties in terms of accumulative damage in the structure. In this work MINI-FALSTAFF spectrum loading was generated by Genesis(6) and removed compressive forces. The spectrum loading MINI-FALSTAFF modified is shown in Figure 4.

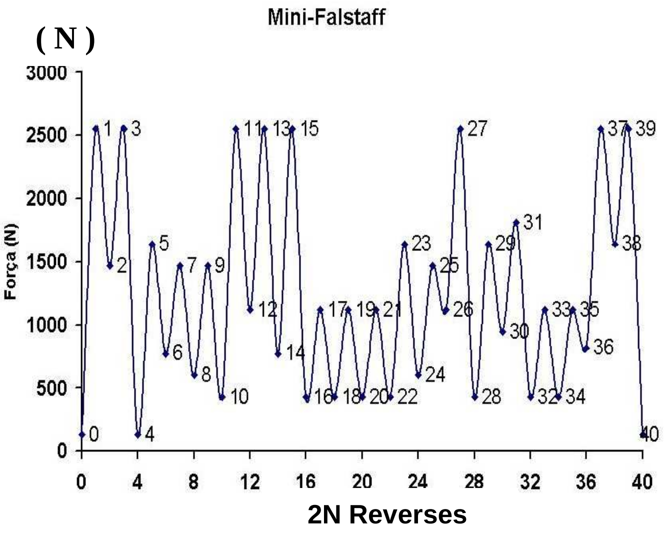

Figure 4. - MINI-FALSTAFF modified.

WISPER is a standard variable-amplitude test loading histories for use in the fatigue design of horizontal axis wind turbine blades. This load history was develop by ten Have(30) and is based on flap load service measurements on 9 different horizontal axis wind turbines, covering a wide range of materials, rotor diameters and geographical locations. This work applies the MINI-WISPER spectrum loading generated by Genesis.(6) Genesis edits the WISPER load story. The spectrum loading used to simulate the crack propagation is a modified wind turbine standard loading MINI-WISPER, having only positive loads, as shown in Figure 5.

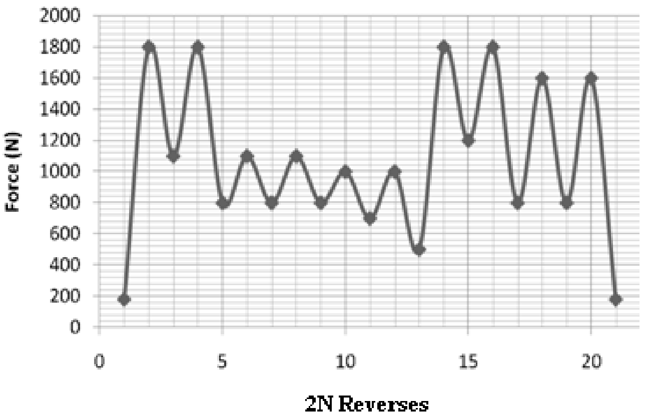

Figure 5. Wind turbine standard loading modified (WISPER).

To convert force in stress intensity factor, the equation (1) is used; fatigue crack growth is simulated by releasing crack tip node. The force is divided into steps of loads P_{min}- P_{max} and nine steps of loads P_{max}-P_{min}, in each cycle.

The smaller element 0.025 mm, was estimated based on the plastic zone size ahead of the crack tip and computed by Irwin equation (2),

$$K_{\min} = \frac{P_{\min}}{BW^{1/2}} f\left(\frac{a}{W}\right); \qquad K_{\max} = \frac{P_{\max}}{BW^{1/2}} f\left(\frac{a}{W}\right) \tag{1}$$

$$r_y = \frac{1}{8\pi}\left(\frac{K_{\max}}{\sigma_y}\right)^2 \text{, (plane stress)} \tag{2}$$

Where: $\boldsymbol{K_{min}}$ = minimum stress intensity factor; $\boldsymbol{K_{max}}$ = maximum stress intensity factor; $\boldsymbol{P_{min}}$ = minimum applied load; $\boldsymbol{P_{max}}$= maximum applied load ; ***B*** = specimen thickness;***a*** = crack length; ***W*** = width of the specimen; ***a/W*** = ratio of the crack length to the specimen width; ***f(a/W)* =** characteristic function of the specimen geometry; $\boldsymbol{r_y}$= cyclic plastic zone size; $\boldsymbol{\sigma_y}$ = effective yield strength. Table I shows the materials properties for compact tension specimen, C(T).

To evaluate the crack propagation, a nonlinear analysis is used to compute the deformation history cycle by cycle, using the Newton-Rapson method. The procedure to estimate if the crack is opened or closed is based on the work of Wei and James.(20) These authors considered that the crack closure occurs at the first contact behind the crack tip; a second criterion is that the surface at the crack tip must be in compression. This can be observed when the displacements of nodes in the crack tip area are negatives in (y) direction. Table 2 display the static mechanical properties of the simulated material, a low alloy steel.

Table 2. Material properties of a low alloy steel

σ_{YS} (MPa)	σ_{TS} (MPa)	E (MPa)	e	ν
230	410	210 000	0.21	0.30

Where: σ_{YS} = yield strength; σ_{TS} = tensile strength; E = Young´s modulus; e = total elongation; ν = Poisson’s ratio

The dimensions of the compact tension specimen were: B=3.8 mm; W= 50.0 mm; a/W= 0.26. Table 3 shows the estimated and used values of the cyclic plastic zone sizes as well as smaller finite element. The smaller element size 0.025 mm, was estimated based on the cyclic plastic zone size ahead of the crack tip and computed by Irwin equation (1). The mentioned values used in Table 3 are choose with intention to improve the quality of results when will be compared with experimental results.

Table 3. Smaller finite element size

	Cyclic Plastic Zone Size	Smaller Finite Element Size
	(mm)	(mm)
Estimated	0.48	0.048
Used	0.10	0.025

The stress level on the crack tip, Figure 6, has to be positive to characterize the crack opening and negative to characterize the crack closure. Antunes and Rodrigues(31) consider two basic points as criteria to determine the crack opening or closure:

– the first contact of the crack flank, which corresponds to the contact of the first node behind the current crack tip. This is the conventional definition proposed by Elber(8) and has been widely used by Jiang et al.(32) However, results are mesh-dependent, since the proximity of the first node to the crack tip increases the opening load;

– the first contact of other nodes behind the crack tip. Roychowdhury and Dodds(33) considered the second node behind the crack tip. In this paper the released nodes in the crack tips were located at the minimum load of a load cycle to simulate crack growth and will be considered the first contact of other nodes behind the crack tip, positive stress (+Syy) to characterize the crack opening and negative stress (-Syy) to characterize the crack closure.

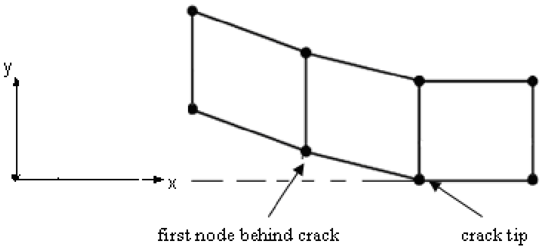

Figure 6. Crack opening and closure criterion.

5 RESULTS

The numerical results of this work are presented in Tables 4 and 5 and will be compared with experimental data. Where: K_{min} = Lower Stress Intensity Factor; K_{max} = Maximum Stress Intensity Factor; K_{op} = Crack Opening Stress Intensity Factor; K_{cl} = Crack Closure Stress Intensity Factor; NI = Not Identified

Table 4. Numerical crack opening and closure data from wisper

Cycle N.o	K_{min} $MPa\sqrt{mm}$	K_{max} $MPa\sqrt{mm}$	K_{op} $MPa\sqrt{mm}$	K_{cl} $MPa\sqrt{mm}$
1	78	255	150	144
2	152	260	140	147
3	112	159	100	90
4	104	162	130	129
5	107	241	145	136
6	144	345	150	195
7	140	226	130	150
8	126	230	130	130
9	149	210	130	118
10	140	213	110	120

Table 5. Numerical crack opening and closure data from mini-Falstaff

Cycle N.o	K_{min}	K_{max}	K_{op}	K_{cl}
	$MPa\sqrt{mm}$	$MPa\sqrt{mm}$	$MPa\sqrt{mm}$	$MPa\sqrt{mm}$
1	90	120	NI	NI
2	10	104	80	70
3	35	90	40	60
4	30	55	33	35
5	31	59	42	33
6	25	61	41	40
7	77	127	90	80
8	39	130	70	84
9	43	120	90	55
10	22	46	30	35

6 DISCUSSION OF RESULTS

In this work, the crack opening and closure were very difficult to determine precisely. It was necessary to use the iteration process in the crack surface step by step during loading and unloading to find the crack opening or closing as did Ricardo.(21) The retard effect is present in some cycles, in special in the presence of overloads. In constant amplitude loading, the effective plastic zone increases with the extension of the crack length; the crack propagation rate has no influence in the quality of results, according with Newman(7) it is recommendation to have four elements yielded in the reverse plastic zone to get reproduce the engineering problem. In variable amplitude loading the crack length can not progress until a new overload occurs or the energy spent during cyclic process creates a new plastic zone and the driving force increases the crack length. The researchers normally work with simple overloads or specific load blocks. This approach can induce some mistakes in terms of results that can be conservative or nonrealistic. The methodology used in this work to simulate the crack propagation is the same as Ricardo(22) under constant amplitude loading, as shown in Figure 7, providing good correlation between numerical and experimental data.

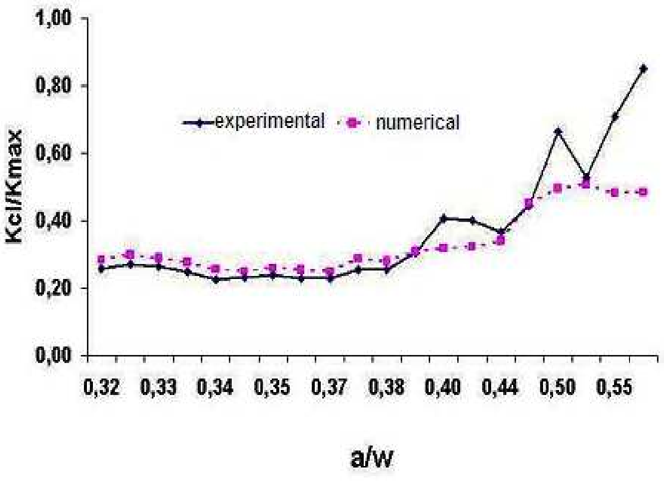

Figure 7. Correlation numerical x experimental data.

It is expected the same level of correlation in this work, while using the same crack propagation mode under variable amplitude loading approach, as Ricardo.(22) It will be necessary to test real crack propagation in specimens to validate the numerical results from the crack propagation model. Of course there are many factors related with the crack propagation rate like reverse and effective plastic zones; this can to be calculated using Irwin expression as well as the numbers of the elements that must have yielded inside the reverse plastic zone.

7 CONCLUSION

In this work the crack opening and closure were possible to identify using finite element method. In the literature there are few works covering crack propagation simulation with random load histories like WISPER and Mini-Falstaff.

Usually, just few load blocks are used to reduce the complexity and this should provide conservative answers when used to develop wind turbine components. The next step in this work is in progress and will be to perform the same model and load history with different crack propagation rates to identify whether or not the retard arrest effect can be observed. These data will be compared with experimental tests and, if necessary, adjustment of the crack propagation model will be done to improve the crack propagation model.

REFERENCES

1 Miner, M. A., Cumulative Damage in Fatigue, Journal of Applied Mechanics, *ASME*, Vol.12, USA, 1945 p. A159-A164.
2 Schijve, J., Fatigue Crack Propagation in Light Alloy Sheet Material and Structures, *NLR*, Report MP195, Amsterdam, 1960
3 Ditlevsen, O. & Sobczyk, K. ;Random Fatigue Crack Growth with Retardation; Eng. Fracture Mech.;Vol.24; N.o 6, pp. 861-878, 1986
4 Dugdale, D.S., Yielding of Steel Sheets Containing Slits, Journal Mech. Phys. Solids, pp. 100-104, n.0 8, USA, 1960
5 Skorupa, M & Skorupa, A., Experimental Results and Predictions on Fatigue Crack Growth in Structural Steel, Int. Journal of Fatigue, 27, pp. 1016-1028, 2005
6 Genesis, Version 1.0, *NRL*, National Research Laboratory, Amsterdam, 2001
7 Newman JC Jr. *Finite-element analysis of fatigue crack propagation including the effects of crack closure*. PhD Thesis, VPI & SU, Blacksburg, VA, 1974
8 Elber W. Fatigue crack closure under cyclic tension, Eng Fracture Mech., Vol.2, pp.37–45,1970
9 Blom AF, Holm DK. An experimental and numerical study of crack closure. Eng. Fract. Mech. 22: pp. 997-1011,1985;
10 Newman, J .C. *A;* Finite Element Analysis of Fatigue Crack Closure, *NASA TM X-72005*, National Aeronautics and Space Administration, Hampton,VA, 1975
11 de Matos PFP, Nowell D. On the accurate assessment of crack opening and closing stresses in plasticity-induced crack closure problems. Eng. Fract. Mech.,74(10):pp.1579–1601,2007
12 Sander, M. & Richard, H. A. ; Lifetime predictions for real loading situations—concepts and experimental results of fatigue crack growth, Int. Journal of Fatigue**,** Volume 25, Issues 9-11, September-November, pp. 999-1005, 2003
13 Edwards PR, Darts J. Standardised fatigue loading sequences for helicopter rotors (HELIX and FELIX). RAE-reports TR 84084 and TR 84085. Royal Aircraft Establishment; 1984.

14 McClung RC, Sehitoglu H. On the finite element analysis of fatigue crack closure 1. Basic modeling issues. Eng. Fract. Mech.;33(2):pp.237–252, 1989

15 McClung RC, Sehitoglu H. On the finite element analysis of fatigue crack closure 2. Numerical results. Eng. Fract. Mech.;33(2):pp.253–272, 1989

16 Sehitoglu Huseyin, Sun Wei. Modelling of plane strain fatigue crack closure. J Eng Mater Technol;113:pp.31–40, 1991

17 Sun Wei, Sehitoglu Huseyin. Residual stress fields during fatigue crack growth. Fatig. Fract. Eng. Mater. Struct.;15(2):pp.115–128, 1992

18 Wu J, Ellyin F. A study of fatigue crack closure by elastic-plastic finite element analysis for constant-amplitude loading. Int. J. Fract. ;82:pp.43–65, 1996

19 Ellyin F, Wu J. A numerical investigation on the effect of an overload on fatigue crack opening and closure behaviour. Fatig. Fract. Eng. Mater. Struct.;22: pp.835–847,1999

20 Wei LW, James MN. A study of fatigue crack closure in polycarbonate ct specimens. Eng. Fract. Mech.;66: pp.223–242, 2000

21 Ricardo, L. C. H ; Pimenta, P. M.; Spinelli D. & Andrade, A H.P. "*Crack Closure Simulation by Finite Element Method",*; In : Blom, A.F (Ed.), Fatigue 2002, Emas Publishing, Stockholm, Sweden, Vol. 4, pp.2863-2869, 2002

22 Ricardo, L. C. H ; *"Modeling Fatigue Crack Opening and Closing Phenomenon by Finite Element Method* ", PhD Thesis, Department of Structures and Foundations, University of Sao Paulo (In portuguese), 2003

23 Solanki K, Daniewicz SR, Newman Jr JC. Finite element modelling of plasticity-induced crack closure with emphasis on geometry and mesh refinement effects. Eng. Fract. Mech.;70:pp.1475–1489, 2003

24 Corbly, D.M. & Packman, P.F. " On The Influence of Single and Multiple Peak Overloads on Fatigue Crack Propagation in 7075-T6511 Aluminum" , Eng. Fract. Mech. 5, pp. 479-497, 1973

25 Schijve, J. ; Brock, D. & de Rigle, P., Report M2094, Amsterdam, 1962

26 Hardrarth, H.F. & McEvily, A . T., Proc. Crack Propagation Symposium, Vol. 1 Cranfield, 1961

27 Jonds, D. & Wei, R.P. , An Exploratory Study of Delay in Fatigue Crack Growth, Eng. Fract. Mech. 7,1971

28 MSC/Patran r1, USA, 2008

29 Abaqus version 68, USA, 2008

30 ten Have, A.A. WISPER and WISPERX - Final definition of two standard fatigue loading sequences for wind turbine blades, NLR TP 91476 U, 1991

31 F.V. Antunes, & Rodrigues, D.M, Numerical Simulation of Plasticity Induced Crack Closure: Identification and Discussion of Parameters, *Eng. Fracture Mech.*, Vol. 75, pp. 3101-3120, 2008

32 Jiang Y, Feng M, Ding F., A re-examination of plasticity-induced crack closure in fatigue crack propagation, *Int. J Plasticity*,21, pp. 1720–40, 2005

33 Roychowdhury S, Dodds Jr RH. A numerical investigation of 3D small-scale yielding fatigue crack growth. *Eng. Fracture Mech*.;Vol. 70, pp.2363–83, 2003