ON THE NOTCH GEOMETRY DEPENDENCE OF THE FATIGUE GROWTH THRESHOLD OF SHORT CRACKS[1]

Marco Antonio Meggiolaro[2]
Antonio Carlos de Oliveira Miranda[3]
Jaime Tupiassú Pinho de Castro[4]

Abstract

Several expressions have been proposed to model the dependency between ΔK_{th} and the crack size a. Most of these expressions are based on length parameters, resulting in a modified stress intensity range. These equations can be used to calculate the notch sensitivity factor q, which can be associated with the presence of non-propagating cracks. In this work, new estimates are proposed to evaluate the growth threshold of short cracks emanating from notches. Topper et al.'s classical approach can easily generate approximate expressions by studying the limit cases where the crack is much smaller or much larger than the notch dimensions. However, short crack behavior is found to be extremely sensitive to the choice of the threshold estimate as a function of crack size. A generalized version of El Haddad-Topper-Smith's equation, based on Bazant's work, is used here to model the threshold stress intensity factor of short cracks emanating from semi-elliptical notches. The results are compared with Finite Element calculations.
Key words: Short cracks; Notch sensitivity; Non-propagating cracks.

SOBRE A DEPENDÊNCIA ENTRE A GEOMETRIA DO ENTALHE E O LIMIAR DE PROPAGAÇÃO DE TRINCAS CURTAS DE FADIGA

Resumo

Várias expressões foram propostas para modelar a dependência entre ΔK_{th} e o tamanho de trinca a. A maioria dessas expressões se baseia em parâmetros de comprimento, resultando em fatores de intensidade de tensão modificados. Essas equações podem ser usadas para calcular o fator de sensibilidade ao entalhe q, que pode ser associado à presença de trincas não-propagantes. Neste trabalho, novas estimativas são propostas para avaliar o limiar de propagação de trincas curtas emanando de entalhes. O enfoque clássico de Topper et al. pode facilmente gerar expressões aproximadas, estudando os casos limites onde a trinca é muito menor ou muito maior do que as dimensões do entalhe. No entanto, o comportamento destas trincas é muito sensível à escolha de estimativas para o limiar de propagação em função do tamanho da trinca. Uma versão generalizada da equação de El Haddad-Topper-Smith, baseada no trabalho de Bazant, é usada aqui para modelar o limiar de propagação de trincas curtas emanando de entalhes semi-elípticos. Os resultados são comparados com cálculos de Elementos Finitos.
Palavras-chave: Trincas curtas; Sensibilidade ao entalhe; Trincas não propagantes.

[1] *Contribuição técnica ao 62° Congresso Anual da ABM – Internacional, 23 a 27 de julho de 2007, Vitória – ES, Brasil.*
[2] *Mechanical Engineer, Ph.D., Professor Dept. Mechanical Eng., PUC-Rio*
[3] *Civil Engineer, Ph.D., Visiting Professor Dept. Mechanical Eng., PUC-Rio*
[4] *Mechanical Engineer, Ph.D., Professor Dept. Mechanical Eng., PUC-Rio*

1 INTRODUCTION
2

The empirical notch sensitivity factor q, widely used in the classical SN design methodology, is caused by small non-propagating fatigue cracks found at notch roots when $\Delta\sigma_0/K_t < \Delta\sigma_n < \Delta\sigma_0/K_f$, where $\Delta\sigma_n$ is the nominal stress range, $\Delta\sigma_0$ is the fatigue limit, K_t is the geometric and K_f is the fatigue stress concentration factors of the notch. Therefore, it should be possible to analitically predict q values based on the propagation behavior of small cracks emanating from notches.

Several expressions have been proposed to model the influence of the size a of very small fatigue cracks on their stress intensity range propagation threshold value, $\Delta K_{th}(a)$.[1] Most of these expressions are based on length parameters such as El Haddad-Topper-Smith's a_0,[2] estimated from ΔK_0, the $\Delta K_{th}(a \to \infty)$ of long cracks and $\Delta\sigma_0$, resulting in a modified stress intensity range

$$\Delta K_I = \Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{1}$$

These equations reproduce the Kitagawa-Takahashi plot trend,[3] one of the most used tools to qualitatively understand the behavior of short cracks, as well as to design for infinite life. A very good review of near-threshold fatigue can be seen in Lawson, Chen e Meshii.[4] Yu *et al.*[5] and Atzori *et al.*[6] used a geometry factor α to generalize the above equation to other geometries, resulting in

$$\Delta K_I = \alpha \cdot \Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\alpha \cdot \Delta\sigma_0}\right)^2 \tag{2}$$

Note however that, in the above expression, a very small crack with $a \ll a_0$ would imply that $\Delta\sigma$ tends to the fatigue limit $\Delta\sigma_0$. In the presence of notches, this would be true only if $\Delta\sigma$ is the notch root stress range, not the nominal one. But in many cases the geometry factor α defined in the literature already includes the effects of the notch root stress concentration factor, defining $\Delta\sigma$ as the nominal stress. To avoid this problem, perhaps a clearer way to define the length parameter a_0 in the presence of notches is by considering $\Delta\sigma$ as the nominal stress range (away from the notch) and two factors $f(a)$ and α, where the former tends to the notch root stress concentration factor as a tends to zero, and the latter only encompasses the remaining terms, such as the free surface correction:

$$\Delta K_I = \alpha \cdot f(a) \cdot \Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\alpha \cdot \Delta\sigma_0}\right)^2 \tag{3}$$

Note that $f(a)$ does not appear in the expression of a_0, because for very small cracks ($a \to 0$) the notch root stress range $f(0)\cdot\Delta\sigma$ should be equal to and replaced by $\Delta\sigma_0$.

Ciavarella and Monno[7] have used length parameters such as the ones presented above to design not only for infinite life, but also for finite lives using an interpolation between the Basquin/Wöhler equations and the Paris law, with or without corrections for the near-threshold ΔK regime. Their resulting expressions can be seen as SN curves which are a function of the initial (small) crack size.

Alternatively, the stress intensity range can retain its original equation,[8-13] while the threshold expression is modified by a function of the crack length a, namely $\Delta K_{th}(a)$, resulting in

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \sqrt{\frac{a}{a + a_0}} \quad (4)$$

Peterson-like[14] expressions can then be calibrated to q based on these crack propagation estimates. Topper's[2] classical approach can easily generate approximate expressions by studying the limit cases where the crack is much smaller or much larger than the notch dimensions. For instance, the stress intensity range of a semi-elliptical notch with stress concentration factor K_t and semi-axis b parallel to a crack with length a, in an infinite plate under tension, has limit values

$$\Delta K_{\mathrm{I}} = 1.1215 \cdot K_t \cdot \Delta\sigma\sqrt{\pi(a + a_0)} \text{ for a << b} \quad (5)$$

$$\Delta K_{\mathrm{I}} = 1.1215 \cdot \Delta\sigma\sqrt{\pi(a + b)} \text{ for a >> b} \quad (6)$$

The threshold stress range $\Delta\sigma_{th}$ necessary to propagate a crack of size a from such notch is then

$$\Delta\sigma_{th} = \frac{\Delta K_0}{\alpha \cdot \sqrt{\pi(a + a_0)}} \leq \frac{1.12 \cdot K_t \cdot \Delta\sigma_0\sqrt{\pi a_0}}{1.12 \cdot \sqrt{(a + b)/a} \cdot \sqrt{\pi(a + a_0)}} \quad (7)$$

The above expression is upper-bounded by

$$\Delta\sigma_{th} \leq K_t \cdot \Delta\sigma_0 \cdot \sqrt{\frac{a}{a + b} \cdot \frac{a_0}{a + a_0}} \quad (8)$$

which is maximum for a critical size $a = \sqrt{b \cdot a_0}$. Therefore, this critical size proposed by Topper is often associated with the maximum $\Delta\sigma_{th}$ for sharp notches, as well as the size of the largest non-propagating crack. However, approximations such as the one presented above are found to be extremely sensitive to the choice of $\Delta K_{th}(a)$ estimate.

In the following section, a generalization of El Haddad-Topper-Smith's equation is used to better model the crack size dependence of $\Delta K_{th}(a)$. This expression is then applied to cracks emanating from circular holes and semi-elliptical notches, resulting in improved estimates of the notch sensitivity q and the largest non-propagating crack size.

2 PROPAGATION OF SHORT CRACKS

The El Haddad-Topper-Smith's equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's reasoning,[15] a more general equation can be proposed, involving a fitting parameter n, which can be written as

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \left[1 + \left(\frac{a_0}{a}\right)^{n/2}\right]^{-1/n} \quad (9)$$

In the above equation, n is typically found to be between 1.5 and 8.0. Clearly, Eqs. (1) through (4) are obtained from Eq. (5) when n = 2.0. Also, the bi-linear estimate is obtained as n tends to infinity. The adjustable parameter n allows the ΔK_{th} estimates to better correlate with experimental crack propagation data collected from Tanaka et al.[16] and Livieri and Tovo [17], see Fig. 1.

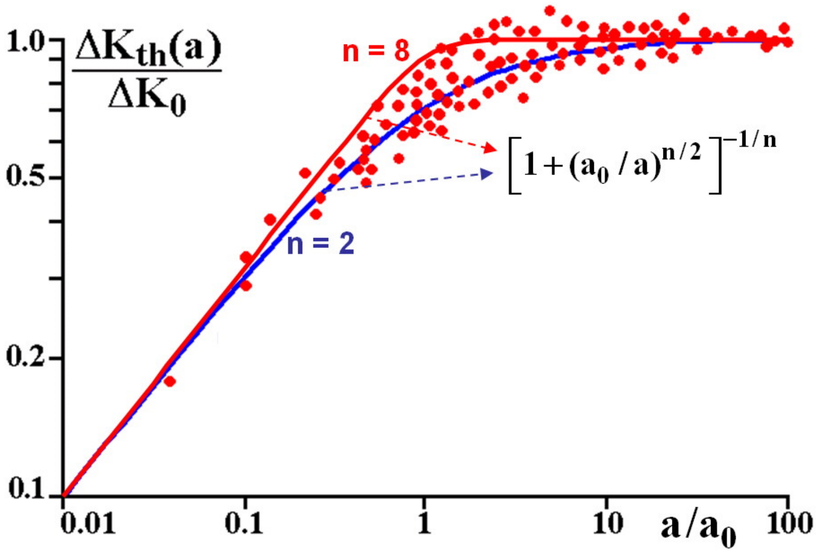

Figure 1 – Ratio between short and long crack propagation thresholds as a function of a/a_0.

Equation (9) is now used to evaluate the behavior of short cracks emanating from semi-elliptical notches. The stress intensity range of a single crack with length *a* emanating from a semi-elliptical notch with semi-axes *b* and *c* (where *b* is in the same direction as *a*) can be written as

$$\Delta K_I = \alpha \cdot f\left(\frac{a}{b},\frac{c}{b}\right)\cdot \Delta\sigma\sqrt{\pi a} \tag{10}$$

where α = 1.1215 is the free surface correction, and $f(a/b, c/b)$ is a geometry factor associated with the notch stress concentration. The geometry factor can be expressed as a function of the dimensionless parameter $s = a / (b + a)$ and the notch root stress concentration factor K_t

$$K_t = \left(1+2\frac{b}{c}\right)\cdot\left[1+\frac{0.1215}{(1+c/b)^{2.5}}\right] \tag{11}$$

Values for *f* have been calculated by Nishitani and Tada,[18] with results available only in graph form, however equations are necessary to perform the analyses. To obtain expressions for *f*, Finite Element (FE) calculations were performed using the Quebra2D program[19] considering several cracked semi-elliptical notch configurations. The numerical results, which agreed well with Nishitani and Tada,[18] were fitted within 3% using empirical equations, resulting in

$$f\left(\frac{a}{b},\frac{c}{b}\right) \equiv f(K_t,s) = K_t\cdot\sqrt{\frac{1-exp(-K_t^2\cdot s)}{K_t^2\cdot s}} \quad \text{for } c \le b \tag{12}$$

$$f\left(\frac{a}{b},\frac{c}{b}\right) \equiv f(K_t,s) = K_t\cdot\sqrt{\frac{1-exp(-K_t^2\cdot s)}{K_t^2\cdot s}}\cdot\left[1-exp(-K_t^2)\right]^{-s/2} \quad \text{for } c \ge b \tag{13}$$

Figures 2 and 3 compare the proposed equations (solid lines) with the FE calculations.

Using Equations (9) through (13), the notch sensitivity of semi-elliptical notches can be obtained. The results are presented next.

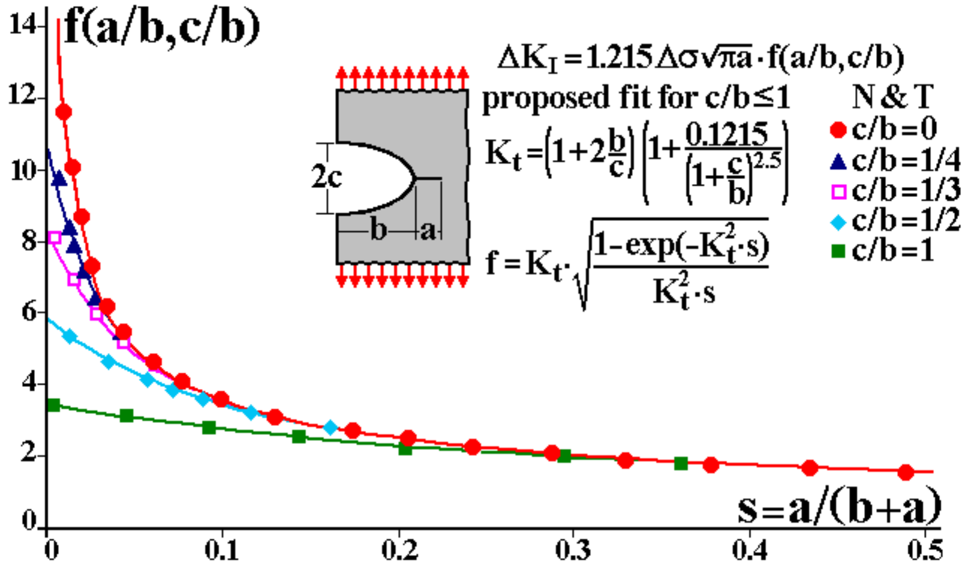

Figure 2 – Finite Element calculations and proposed fit for the geometry factor of semi-elliptical notches with c ≤ b.

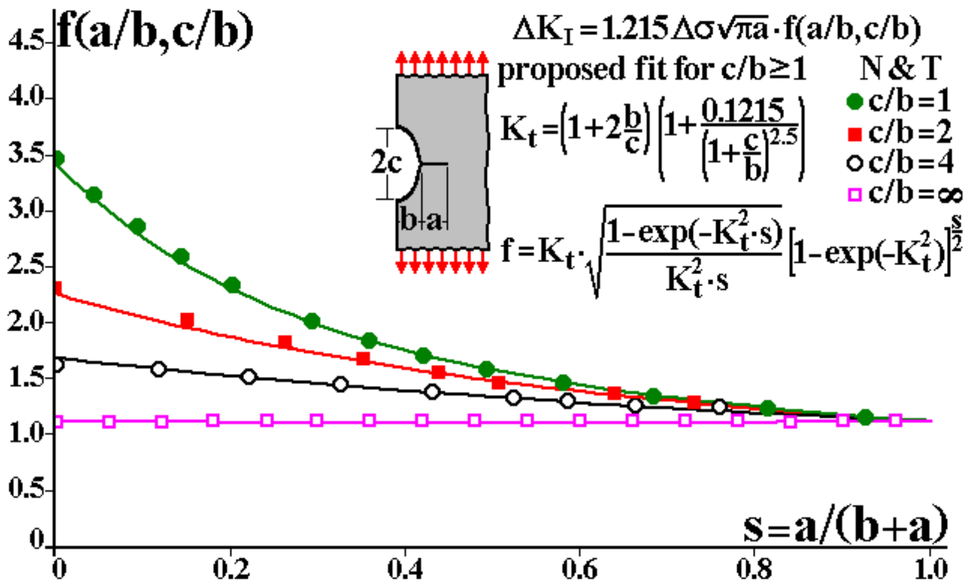

Figure 3 – Finite Element calculations and proposed fit for the geometry factor of semi-elliptical notches with c ≥ b.

3 RESULTS

For several combinations of k and n, the smallest stress range necessary to both initiate and propagate a crack from semi-elliptical notches is calculated, resulting in expressions for K_f and therefore q. As expected, the results depend on ρ, $\Delta\sigma_0$ and ΔK_0, in addition to n. Moreover, a significant dependency is observed with respect to the aspect ratio c/b. Therefore, the entire notch geometry, not only its radius, is an important factor when evaluating its sensitivity.

Figures 4 and 5 show the calculated notch sensitivity q as a function of the semi-elliptical notch root radius ρ for several aspect ratios c/b. These figures show representative values for aluminum alloys with S_u near 225MPa, selected from ViDa [19], as well as steels with S_u near 800MPa. The associated crack length parameters according to Eq. (3) are, respectively, a_0 = 0.26mm and 0.10mm, averaged from the selected sample. It can be seen that *q* has a strong dependence on the notch geometry through the *c/b* ratio.

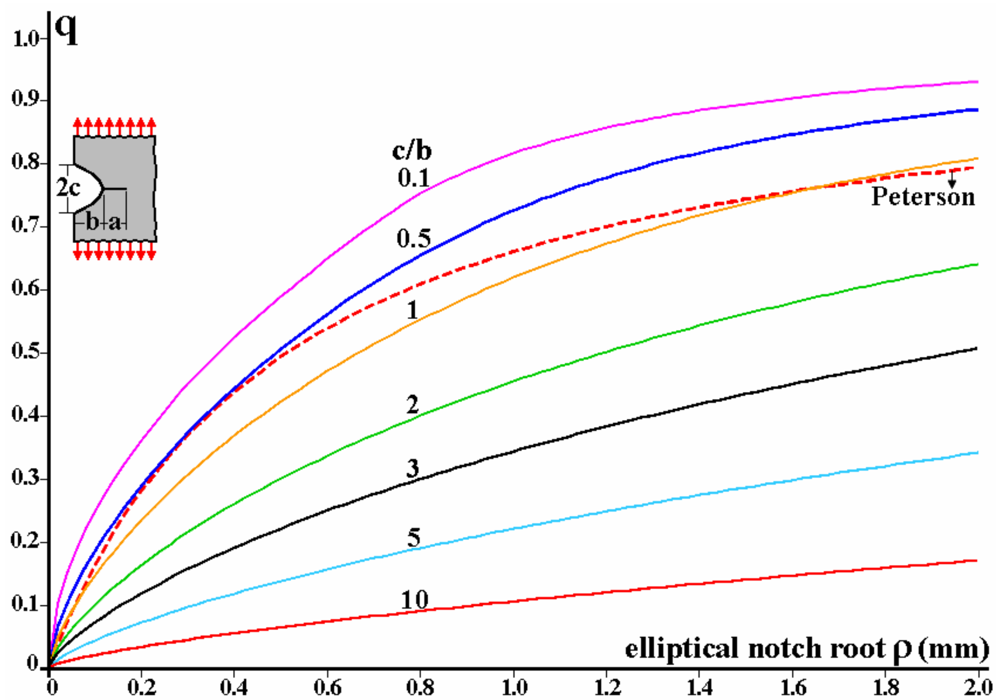

Figure 4 - Notch sensitivity *q* as a function of the semi-elliptical notch root radius ρ for aluminum alloys with a_0 = 0.26mm ($S_u \cong$ 225MPa).

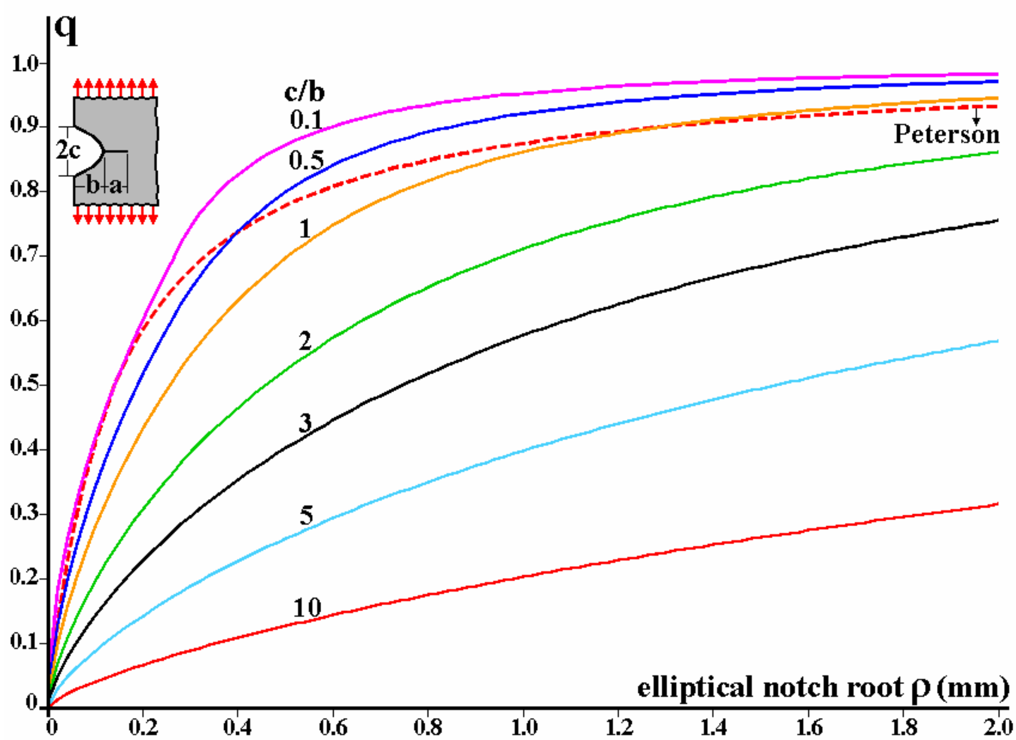

Figure 5 - Notch sensitivity *q* as a function of ρ for steels with a_0 = 0.10mm ($S_u \cong$ 800MPa).

For semi-elliptical notches with larger aspect ratios (*c/b* between 2 and 10), another interesting dependence is found, with the square root of the product between the semi-axes, see Figures 6 and 7. This dependence is in agreement with Murakami's factor,[20] which states that the notch sensitivity associated with internal defects depends on the square root of the area. For *c/b* ratios outside this range, however, there's a significant influence of *c/b* in the resulting *q* values.

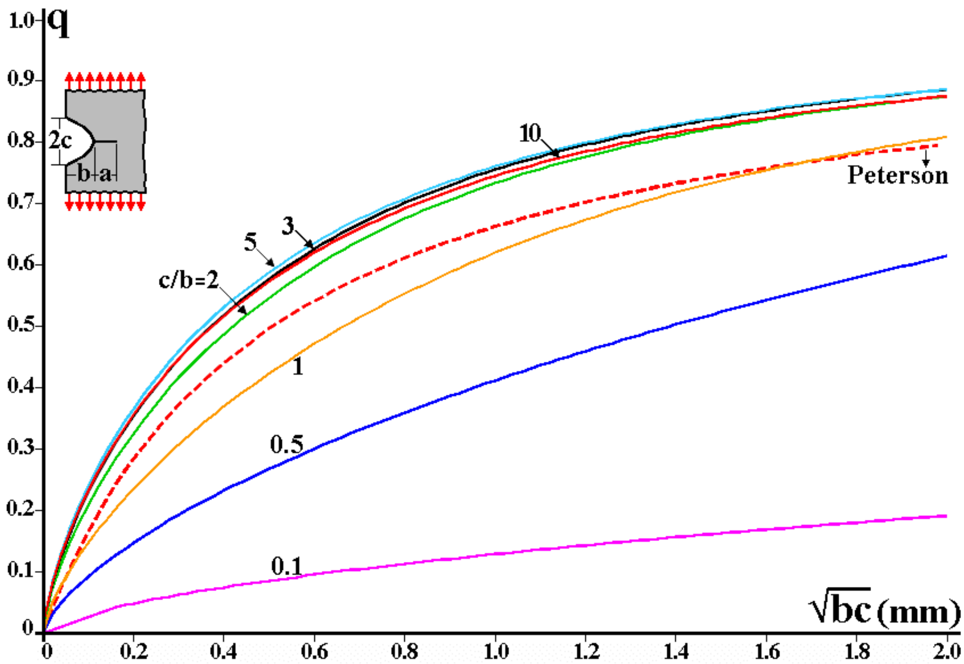

Figure 6 - Notch sensitivity *q* as a function of the square root of the product between the semi-axes for aluminium alloys with a_0 = 0.26mm ($S_u \cong$ 225MPa).

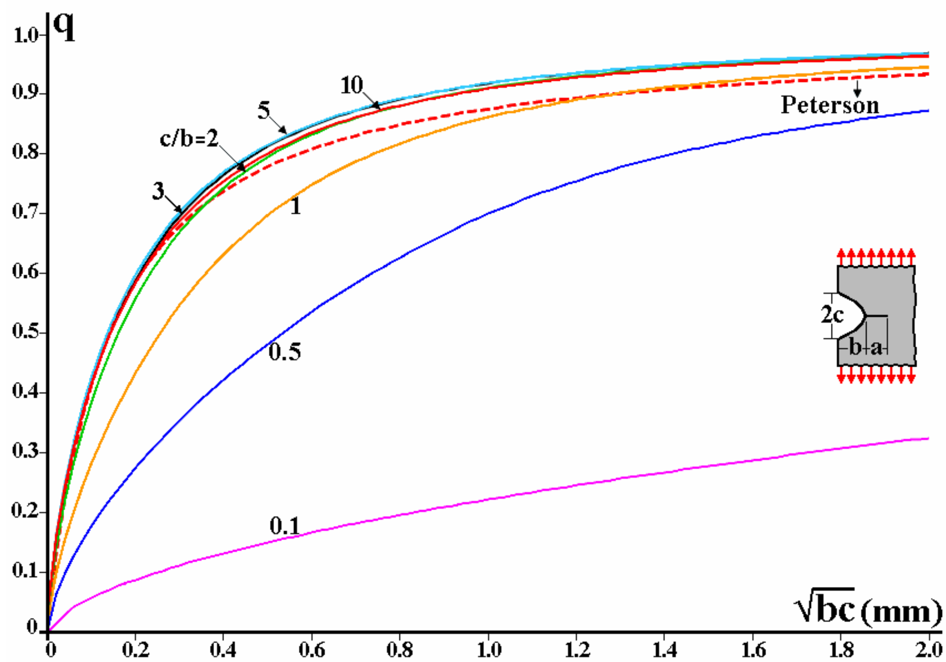

Figure 7 - Notch sensitivity *q* as a function of the square root of the product between the semi-axes for steels with a_0 = 0.10mm ($S_u \cong$ 800MPa).

4 CONCLUSIONS

In this work, expressions were proposed to calculate the behavior of non-propagating cracks. Estimates for the notch sensitivity factor were obtained and compared with experimental results from the literature. It was found that notch sensitivity estimates vary significantly with the notch geometry, e.g. the aspect ratio of the semi-elliptical notch, and with a material-related exponent. This parameter allows the threshold estimates to better correlate with experimental crack propagation data collected from Tanaka et al. and Livieri and Tovo, reproducing most of the behavior in the Kitagawa-Takahashi plot.

REFERENCES

1 CHAPETTI, M.D., Fatigue propagation threshold of short cracks under constant amplitude loading, Int. J. of Fatigue, v.25, p.1319–1326, 2003.
2 EL HADDAD, M.H., TOPPER, T.H., SMITH, K.N., *Engng. Fract. Mech.*, v.11, p.573-584, 1979.
3 KITAGAWA, H., TAKAHASHI, S., In Proceedings of Second International Conference on Mechanical Behavior of Materials, Boston, MA, ASM, p.627–631, 1976.
4 LAWSON, L., CHEN, E.Y., MESHII, M., Near-threshold fatigue: a review. *Int. J. of Fatigue*, 1999; 21:S15-S34.
5 YU, M.T., DUQUESNAY, D.L., TOPPER, T.H., Notch fatigue behavior of 1045 steel, *Int. J. of Fatigue*, 1988;10:109-116.
6 ATZORI, B., LAZZARIN, P., MENEGHETTI, G., Fracture Mechanics and Notch Sensitivity, *Fatigue Fract. Engng. Mater.Struct.*, v.26, p.257-267, 2003.
7 CIAVARELLA, M., MONNO, F., On the possible generalizations of the Kitagawa-Takahashi diagram and of the El Haddad equation to finite life. *Int. J. of Fatigue*, 2006:in press.
8 SADANANDA, K., VASUDEVAN, A.K., Short crack growth and internal stresses, Int. J. Fatigue, v.19, Supp. 1, p.S99–S108, 1997.
9 VALLELLANO, C., NAVARRO, A., DOMINGUEZ, J., Fatigue crack growth threshold conditions at notches. Part I: theory, Fatigue Fract Engng Mater Struct, v.23, p.113-121, 2000.
10 ISHIHARA, S., MCEVILY, A.J., Analysis of short fatigue crack growth in cast aluminum alloys, International Journal of Fatigue, v.24, p.1169–1174, 2002.
11 CIAVARELLA, M., MENEGHETTI, G., On fatigue limit in the presence of notches: classical vs. recent unified formulations. *Int. J. of Fatigue*, 2004;26:289-298.
12 ABDEL-RAOUF, H., TOPPER, T.H., PLUMTREE, A., A model for the fatigue limit and short crack behaviour related to surface strain redistribution. *Fatigue Fract. Eng. Mater. Struct.* 1992; 15(9):895-909.
13 DU QUESNAY, D.L., YU, M.T., TOPPER, T.H., An analysis of notch-size effects at the fatigue limit. J. of Testing and Evaluation, 1988;16(4):375-385.
14 PETERSON, R.E., Stress Concentration Factors, Wiley 1974
15 BAZANT, Z.P., Scaling of quasibrittle fracture: asymptotic analysis. *Int. J. of Fracture*, 1997; 83(1):19-40.
16 TANAKA, K., NAKAI, Y., YAMASHITA, M., Fatigue growth threshold of smal cracks, *Int. J. of Fracture*, 1981;17:519-533.

17 LIVIERI, P., TOVO, R., Fatigue limit evaluation of notches, small cracks and defects: an engineering approach, *Fatigue Fract. Engng. Mater.Struct.*, v.27, p.1037-1049, 2004.
18 TADA, H., PARIS, P.C., IRWIN, G.R., The Stress Analysis of Cracks Handbook, Del Research, 1985.
19 MIRANDA, A.C.O., MEGGIOLARO, M.A., CASTRO, J.T.P., MARTHA, L.F., BITTENCOURT, T.N., Fatigue Crack Propagation under Complex Loading in Arbitrary 2D Geometries, ASTM STP 1411, v.4, p.120-146, 2002.
20 MURAKAMI, Y. Metal Fatigue: Effects of Small Defects and Nonmetallic Inclusions. Elsevier, 2002.