ISSN 1516-392X

PLASTIC ZONE ESTIMATES BASED ON T-STRESS AND ON COMPLETE WESTERGAARD STRESS FIELDS[1]

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Abstract

Real cracks are always accompanied by plastic zones (*pz*) which involve their tips and strongly influence their behavior. Classical estimates for such *pz*, based only on the crack stress intensity factor, are too imprecise for stresses used in practical applications. Improved estimates considering the T-stress, obtained from the Williams series zero order term, do not satisfy boundary conditions, in particular the nominal stresses far from the crack tip, which have a major influence on *pz* size and shape. The complete linear elastic stress field solution for the Griffith plate, obtained by three different methods, is used to prove this affirmative. The first is based on its Westergaard stress function, the second on the equivalent Inglis, and the third on the complete Williams series. Equilibrium corrections necessary to compensate for the stress limitation inside pz provide still better estimates for their sizes and shapes. For more complex structures, for which the geometry and type of loading may also significantly influence *pz* sizes and shapes, the plastic zones can be better estimated from their complete elastic stress field calculated e.g. by finite element procedures.

Key-words: Crack tip plastic zone estimates; T-stress; Equilibrium corrections.

ESTIMATIVAS DE ZONAS PLÁSTICAS BASEADAS EM T-STRESS E EM CAMPOS COMPLETOS DE WESTERGAARD

Resumo

Trincas reais são sempre acompanhadas por zonas plásticas (*zp*) que envolvem suas pontas e influenciam fortemente o seu comportamento. Estimativas clássicas para estas *zp*, baseadas somente no fator de intensidade de tensões, são imprecisas demais para tensões usadas em aplicações práticas. Estimativas melhoradas que consideram as T-*stress*, obtidas do termo de ordem zero da série de Williams, não satisfazem as condições de contorno, em particular a tensão nominal longe da ponta da trinca, que influencia muito o tamanho e a forma da *zp*. A solução completa para o campo de tensões linear elástico na placa de Griffith, obtida por três métodos diferentes, é usada para provar esta afirmativa. A primeira é baseada na sua função de tensão de Westergaard, a segunda na placa de Inglis equivalente, e a terceira na série completa de Williams. Correções de equilíbrio necessárias para compensar a limitação das tensões dentro da *zp* geram estimativas ainda melhores para os tamanhos e as formas das *zp*. Para estruturas mais complexas, nas quais a geometria e o tipo de carga também podem afetar muito as *zp*, elas podem ser mais bem estimadas a partir dos seus campos de tensão linear elásticos completos, calculados e.g. por elementos finitos.

Palavras-chave: estimativas de zonas plásticas, T-stress; correções de equilíbrio

[1] Contribuição técnica ao 66º Congresso Anual da ABM, 18 a 22 de julho de 2011, São Paulo, SP, Brasil.

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1 INTRODUCTION

The stress intensity factor (SIF) alone cannot model well some simple crack problems. E.g. the linear elastic (LE) stress field generated by a SIF $K_I = \sigma_n\sqrt{\pi a}$ in a Griffith plate with a $2a$ crack, loaded in mode I by a nominal stress σ_n, does not obey the boundary conditions far from the crack tip: $\sigma_{ij} = [K_I/\sqrt{(2\pi r)}]\cdot f_{ij}(\theta) \Rightarrow \sigma_\theta(r \to \infty, 0) = 0$, instead of $\sigma_\theta(r \to \infty, 0) = \sigma_n$ as it should, where r is the distance from the tip, θ is the angle measured from the crack plane and $f_{ij}(\theta)$ are the Irwin (or Williams) θ-functions. Since SIF-based fields are LE, they obviously cannot describe stresses and strains inside the plastic zones $pz(\theta)$ which always involve the crack tips either.

Plastic zones sizes are very important for practical applications because they control the applicability of Linear Elastic Fracture Mechanics concepts. Even the most elementary introductions to fracture problems mention that such concepts can only be used if the plastic zone size is much smaller than the cracked structural component dimensions, a concept that requires the pz size to start with. But such sizes cannot be precisely calculated by practical means. Both for teaching and designing purposes, $pz(\theta)$ are traditionally estimated from simplified LE analysis, assuming they depend only on K_I (in the cases the crack is loaded in pure mode I). Indeed, equating the LE Mises stress to S_Y, the yielding strength, the simplest mode I elastic-plastic frontiers in plane stress (pl-σ) and in plane strain (pl-ε) are estimated by Unger:[1]

$$\begin{aligned} pz(\theta)_{pl-\sigma} &= (K_I^2/2\pi S_Y^2)\cdot\cos^2(\theta/2)\cdot[1+3\sin^2(\theta/2)] \\ pz(\theta)_{pl-\varepsilon} &= (K_I^2/2\pi S_Y^2)\cdot\cos^2(\theta/2)\cdot[(1-2\nu)^2+3\sin^2(\theta/2)] \end{aligned} \qquad (1)$$

where ν is Poisson's coefficient. Thus, according to this classical estimate, the $pz(\theta)$ size directly ahead of crack tips in pl-σ, the reference used here to normalize pz plots, should be $pz(0)_{pl\text{-}\sigma} = pz_0 = (1/2\pi)(K_I/S_Y)^2$. But the $\sigma_{ij} = f(K_I)$ hypothesis is exact only when $r \to 0$, or exactly where the assumed LE behavior is singular, thus has no physical sense. It is important to emphasize that singular elastic-plastic (EP) estimates, such as the HRR field, do not solve this problem either, and also generated but approximated pz frontiers. As the pz border may not be too close to crack tips, it is worth to at least estimate the effect of σ_n/S_Y on $pz(\theta)$, where S_Y is the yielding strength, instead of simply neglecting it. A simplistic but clear estimate for this σ_n/S_Y effect can be made forcing $\sigma_y(x\to\infty, y=0) = \sigma_n$, by adding up a constant stress $\sigma_y = \sigma_n$ to the Williams (or Irwin) stress LE field to obtain

$$\sigma(\theta)_{M,pl-\sigma}^{Wil+\sigma_n} = [(\kappa f_x)^2 + (\kappa f_y+\sigma_n)^2 - (\kappa f_x)(\kappa f_y+\sigma_n) + 3(\kappa f_{xy})^2]^{1/2} \qquad (2)$$

where $\sigma(\theta)_{M,pl-\sigma}^{Wil+\sigma_n}$ is the resulting LE Mises stress distribution around the crack tip in pl-σ (considering the σ_n/S_Y effect), $\kappa = K_I/\sqrt{(2\pi r)}$, and $f_x(\theta)$, $f_y(\theta)$, and $f_{xy}(\theta)$ are the mode I θ-functions associated with σ_x, σ_y and τ_{xy}. A similar equation can be easily generated for pl-ε. The corresponding $pz(\theta)$ are obtained from $\sigma_M(\theta) = S_Y$ (Figure 1).

Figure 1 indicates that the σ_n/S_Y ratio may significantly affect $pz(\theta)$ under real loading conditions, since engineering structures are typically designed with yield safety factors $1.2 < \phi_Y < 3$. However, it cannot prove that the σ_n/S_Y effects are that important, since the hypothesis used to generate this plots is not sound. But this simplistic estimate points out that the $pz(\theta)$ dependence on σ_n/S_Y should be further explored, as done in the following sections.

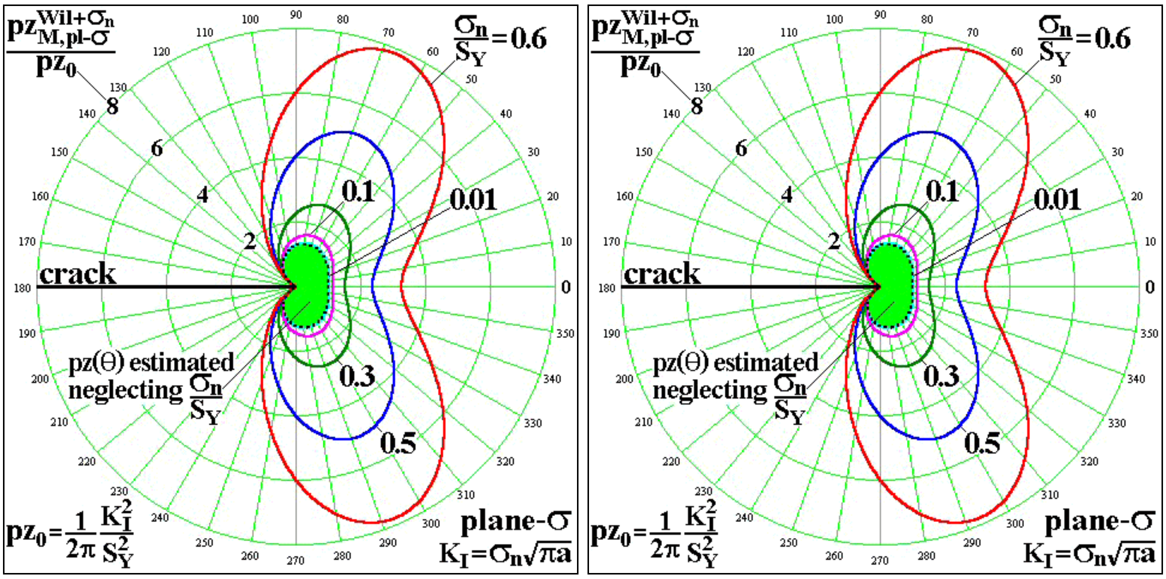


Figure 1: Mode I plastic zones $pz(\theta)$ frontiers roughly estimated for the Griffith plate by $\sigma(\theta)_{M,pl-\sigma}^{Wil+\sigma_n} = S_Y$ for *pl-σ* and for *pl-σ* limit conditions.

2 PLASTIC ZONES ESTIMATED FROM THE INGLIS STRESS FIELD

A much better estimate for the σ_n/S_Y effect on $pz(\theta)$ is obtained from the Inglis plate with a very sharp elliptical notch of major semi-axis a normal to σ_n, and minor semi-axis $b << a$. Making $x = c\cdot\cosh\alpha\cdot\cos\beta$ and $y = c\cdot\sinh\alpha\cdot\sin\beta$, this notch is described in elliptical-hyperbolical coordinates (α,β) by $\alpha = \alpha_0$, where $a = c\cdot\cosh\alpha_0$, $b = c\cdot\sinh\alpha_0$, and $c = a/\cos\alpha_0$. The general solution for the LE stress field σ_α, σ_β, and $\tau_{\alpha\beta}$ in Inglis plates is given by a series too long to be reproduced here.[2,3] But if the very sharp elliptical notch has a tiny (but finite) tip of radius $\rho = b^2/a \cong CTOD/2 = 2K_I^2/\pi S_Y E'$, where $E' = E$ in pl-σ or $E' = E/(1 - \nu^2)$ in pl-ε, then its (LE) stress concentration factor $K_t = 1 + 2a/b$ is given by:

$$K_t = 1 + 2\cdot\frac{a}{b} = 1 + 2\sqrt{\frac{a}{\rho}} = 1 + 2\cdot\sqrt{\frac{a\pi E' S_Y}{2\cdot\sigma_n^2\pi a}} \Rightarrow \frac{a}{b} = \sqrt{\frac{E'}{2\cdot\sigma_n}\cdot\frac{S_Y}{\sigma_n}} = \sqrt{\frac{E'\phi_Y}{2\cdot\sigma_n}} \tag{3}$$

Using this a/b ratio to obtain the ellipsis that emulates the crack in elliptical coordinates by $\alpha_0 = \tanh^{-1}(b/a)$, then the LE stresses in the Inglis plate that simulates the Griffith plate stress field can be calculated. Finally, the Mises stress resulting from σ_α, σ_β, $\tau_{\alpha\beta}$, and $\sigma_z = \nu(\sigma_\alpha + \sigma_\beta)$ can be used to estimate the Inglis plastic zone frontiers $pz(\theta)$ by numerically solving equation (4) for $|\theta| \le \pi$ (Figure 2).

$$\begin{cases} \sigma_{M,pl-\sigma}^{Ing} = \sqrt{\sigma_\alpha^2 + \sigma_\beta^2 - \sigma_\alpha\sigma_\beta + 3\tau_{\alpha\beta}^2} = S_Y \\ \sigma_{M,pl-\varepsilon}^{Ing} = \sqrt{0.5[(\sigma_\alpha - \sigma_\beta)^2 + (\sigma_\alpha - \sigma_z)^2 + (\sigma_z - \sigma_\beta)^2] + 3\tau_{\alpha\beta}^2} = S_Y \end{cases} \tag{4}$$

ISSN 1516-392X

Therefore, the nominal stress influence on these LE estimates for Griffith's plate plastic zone frontiers $pz(\theta)$, although a little less than estimated by the simplistic Figure 1 approximation, is indeed significant and should not be neglected in practical applications. Note that to use Inglis to obtain an exact LE stress field which emulates the Griffith plate in mode I, when the crack is modeled as an elliptical sharp notch of tip radius $\rho = CTOD/2$, is a quite sensible and reasonable hypothesis, since ideal cracks should open by CTOD under load. Nevertheless, it is worth to use an alternative approach to confirm it, as follows.

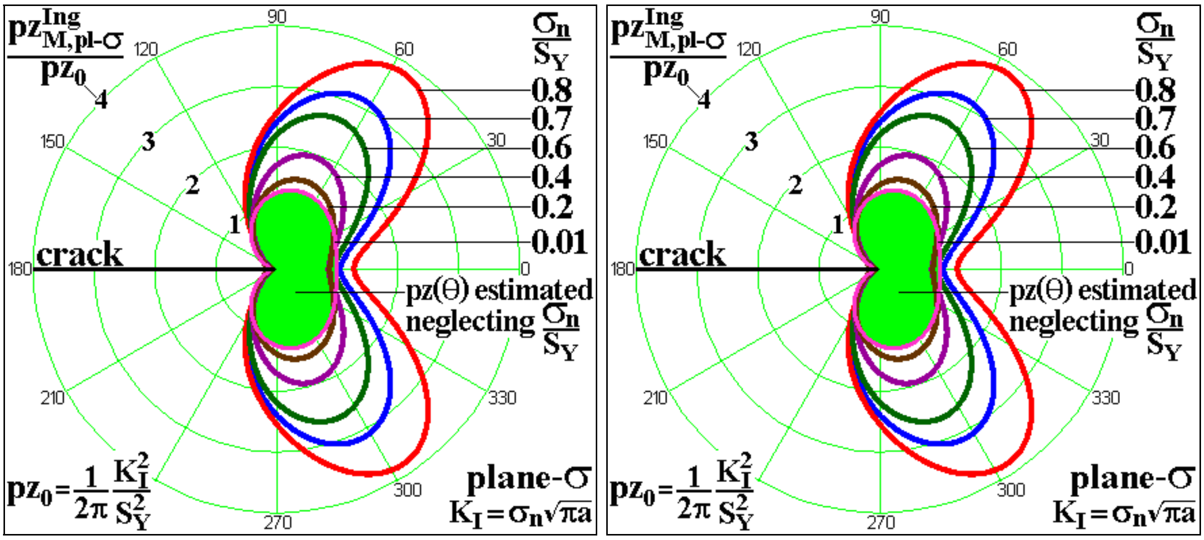


Figure 2: Mises plastic zones $pz(\theta)$ in pl-σ and pl-ε, calculated from the Inglis LE stress field for a cracked plate loaded in mode I, modelling the crack as a very sharp elliptical notch of tip radius $\rho = CTOD/2$.

3 PLASTIC ZONES ESTIMATED BY THE WESTERGAARD STRESS FUNCTION

The Westergaard $Z(z)$ complex stress function provides an alternative way to rigorously estimate the plastic zone frontier $pz(\theta)$ from the Griffith plate elastic stress field.[3,4] But, since the elastic-plastic frontier is not adjacent to the crack tip, the full stresses generated from $Z(z)$ must be used in such a calculation. This is easily demonstrated revisiting the classical Irwin solution for the infinite plate with a crack of size $2a$ loaded in mode I. Thus, if (x, y) and (r, θ) are Cartesian and polar coordinates centered at the crack tip, $i = \sqrt{-1}$ and $z = x + iy$ is a complex variable, the Irwin solution is obtained from the Westergaard stress function

$$Z(z) = z\sigma_n/\sqrt{(z^2 - a^2)} \Rightarrow Z'(z) = dZ/dz = -a^2\sigma_n/(z^2 - a^2)^{3/2} \quad (5)$$

And the corresponding linear stress field is given by

$$\sigma_x = Re(Z) - y\,Im(Z') - \sigma_n, \;\; \sigma_y = Re(Z) + y\,Im(Z'), \;\; \tau_{xy} = -y\,Re(Z') \quad (6)$$

Note that to solve the mode I problem from $Z(z)$ a constant term $-\sigma_n$ has to be added to $\sigma_x = Re(Z) - yIm(Z')$ to force the stress field to obey the Griffith plate boundary condition $\sigma_x(\infty) = 0$, an adequate mathematical trick since a constant stress in the x direction does not affect the LE stress field near the crack tip. It is convenient to rewrite Z and Z' in polar coordinates centered at the crack tip:

$$\begin{cases} Z = \dfrac{[a+(r\cdot cos\,\theta)+i\,(r\cdot sin\,\theta)]\cdot\sigma_n}{\sqrt{[a+(r\cdot cos\,\theta)+i\,(r\cdot sin\,\theta)]^2-a^2}} \\ Z' = \dfrac{-a^2\cdot\sigma_n}{\left\{[a+(r\cdot cos\,\theta)+i\,(r\cdot sin\,\theta)]^2-a^2\right\}^{3/2}} \end{cases} \qquad (7)$$

Substituting (7) in (6), the Mises elastic-plastic frontier $pz(\theta)$ is then given by

$$\left\{\left[Re\left(\frac{(a+r\cdot cos\,\theta+i\cdot rsin\theta)\cdot\sigma_n}{\sqrt{(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2}}\right)-y\,Im\left(\frac{-a^2\cdot\sigma_n}{\left[(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2\right]^{3/2}}\right)-\sigma_n\right]^2+\right.$$
$$+\left[Re\left(\frac{(a+r\cdot cos\,\theta+i\cdot rsin\theta)\cdot\sigma_n}{\sqrt{(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2}}\right)+y\,Im\left(\frac{-a^2\cdot\sigma_n}{\left[(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2\right]^{3/2}}\right)\right]^2-$$
$$-\left[Re\left(\frac{(a+r\cdot cos\,\theta+i\cdot rsin\theta)\cdot\sigma_n}{\sqrt{(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2}}\right)-y\,Im\left(\frac{-a^2\cdot\sigma_n}{\left[(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2\right]^{3/2}}\right)-\sigma_n\right]\cdot \qquad (8)$$
$$\cdot\left[Re\left(\frac{(a+r\cdot cos\,\theta+i\cdot rsin\theta)\cdot\sigma_n}{\sqrt{(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2}}\right)+y\,Im\left(\frac{-a^2\cdot\sigma_n}{\left[(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2\right]^{3/2}}\right)\right]+$$
$$\left.+3\cdot\left[-y\,Re\left(\frac{-a^2\cdot\sigma_n}{\left[(a+r\cdot cos\,\theta+i\cdot rsin\theta)^2-a^2\right]^{3/2}}\right)\right]^2\right\}^{1/2}-S_Y=0$$

Note that the $\sigma y = Re(Z) - yIm(Z')$ stress is usually approximated to generate a SIF (a highly desirable feature but for estimating $pz(\theta)$, since it neglects the σ_n/S_Y effect) by writing

$$\sigma_y\,(\theta=0)=\sigma_n\,(x+a)\Big/\sqrt{(x+a)^2-a^2}\cong\sigma_n a\Big/\sqrt{2ax}=K_I\Big/\sqrt{2\pi r}\text{ , if x << a} \qquad (9)$$

where $2a$ is the crack size perpendicular to the nominal stress σ_n. As (9) formally yields $\sigma_y(\theta=0)=K_I/\sqrt{(2\pi\cdot r)}=0$ if $r\to\infty$, this classical approximation obviously cannot be used to study the σ_n/S_Y influence on $pz(\theta)$. That is why this task must be fulfilled by first calculating the complete stress field generated from Z and Z' to obtain the resulting Mises (or Tresca, for that matter) stress, and then equating it to S_Y to obtain the required $pz(\theta)$ EP frontiers considering the σ_n/S_Y effect, as in equation (8). The same process can be easily applied in pl-ε (Figure 3). Inglis and Westergaard pz visually coincide when the sharp ellipsis has its minor semi-axis (instead of its tip radius) $b = CTOD/2 = 2K_I^2/\pi S_Y E'$ (Figure 4). As $pz_{Ing}(\theta)$ and $pz_{Wtg}(\theta)$ are obtained from completely different equations, their near coincidence is certainly not fortuitous. Therefore, the large σ_n/S_Y effect predicted by these rigorous solutions really should not be neglected in practice. This point must be emphasized for design purposes, since it is the plastic zone size that ***validates*** most LEFM predictions.

ISSN 1516-392X

4 PLASTIC ZONES ESTIMATED FROM THE COMPLETE WILIAMS SERIES

As the Williams series can be used to obtain exact LE solutions for cracked components, its coefficients can then be adjusted to the Griffith plate complete stress field generated from the Westergaard stress function, successively incrementing its number of terms.(5) Figure 5 shows the EP frontiers ahead of the crack tip obtained considering 1 to 4 terms in Williams series. 3 terms are already sufficient to visually reproduce $pz_{Ing}(\theta) = pz_{Wtg}(\theta)$. Thus, exactly as expected, these three paths lead to the same $pz(\theta)$ estimations.

These estimates are based on the Griffith plate correct LE stress field, which obeys the plate boundary conditions (i.e. $\sigma_y(|x| \le a\,,\, y = 0) = \tau_{xy}(|x| \le a\,,\, y = 0) = \sigma_x^{\infty} = \tau_{xy}^{\infty} = 0$, $\sigma_y^{\infty} = \sigma_n$). Thus they are the best $pz(\theta)$ LE estimates that can be obtained for the Griffith plate without considering equilibrium requirements between the applied force and the stresses it generates. However, as the stresses inside the plastic zone are limited by yielding, the truncated LE stress field cannot obey equilibrium conditions. But such conditions can have a major influence on $pz(\theta)$, as recognized by Irwin a long time ago. The next topic considers them, and compares the resulting equilibrated $pz(\theta)$ estimations with $pz(\theta)$ estimated considering only the T-stress correction.

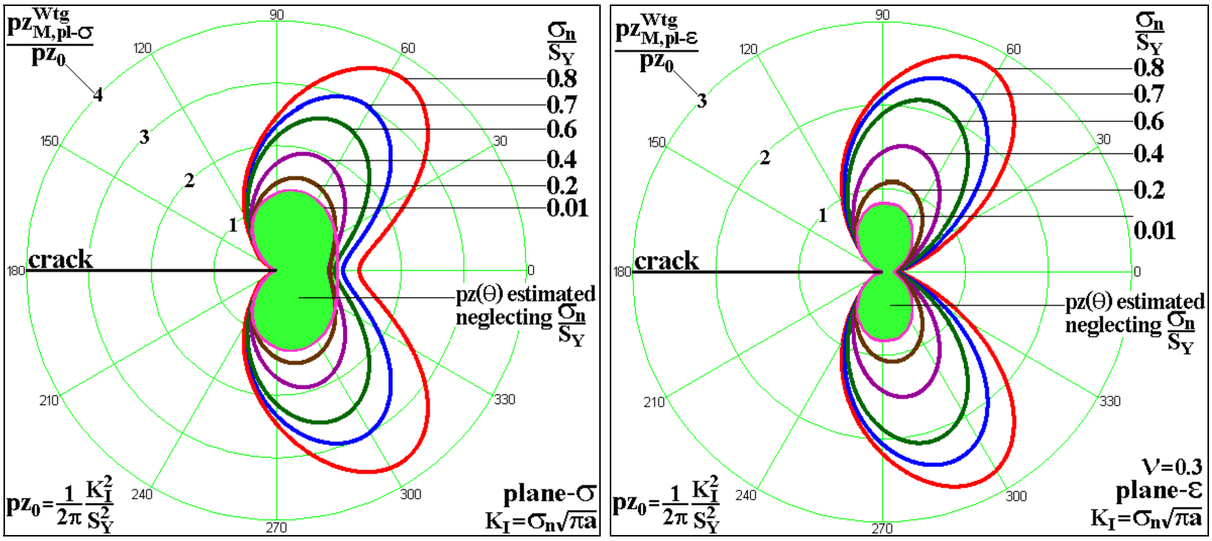


Figure 3: Mises $pz(\theta)$ for the Griffith plate in mode I, estimated from the complete LE stress field induced by the Westergaard stress function for *pl*-σ and *pl*-ε conditions.

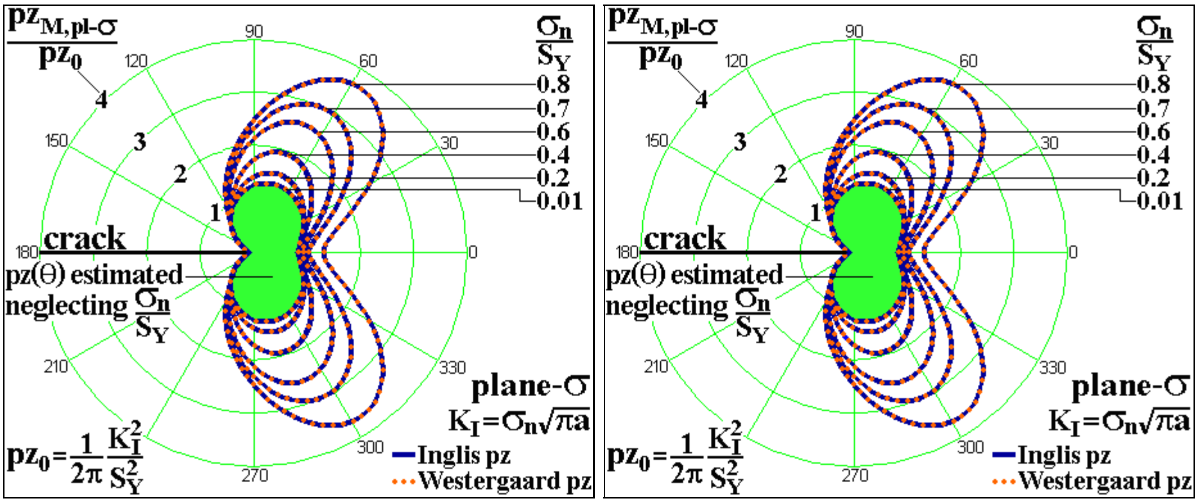
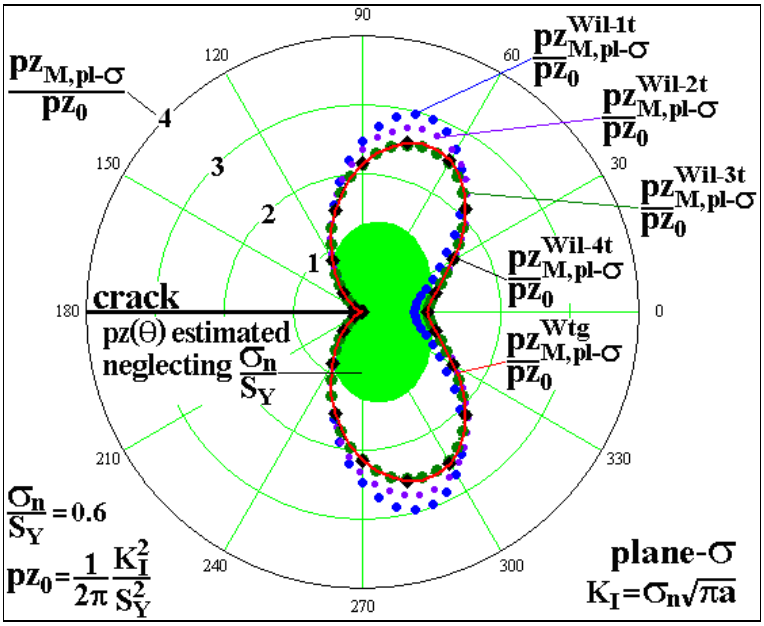


Figure 4: The Mises plastic zone frontiers *pz(θ)* estimated by the complete Westergaard stress field are visually identical to the Inglis estimate when a sharp elliptical notch with $b = CTOD/2 = 2K_I^2/\pi S_Y E'$ instead of $\rho = CTOD/2$ is used to model the crack.

5 T-STRESS AND EQUILIBRIUM INFLUENCE ON pz(θ) ESTIMATIONS

The T-stress correction is a constant σ_x term (parallel to the crack) added to the K_I-based LE stress field which can alleviate some of its limitations.(6) Thus, it has been widely explored in the literature to model some interesting problems.(7-16) From a practical point of view, Fett(17) lists T-stress values for several geometries. However, the resulting K_I +T-stress field cannot reproduce the $\sigma_y^\infty = \sigma_n$ boundary condition in the Griffith plate, as it is just a simplification of the complete LE stress field used above. It is then interesting to compare the plastic zones estimated by them.



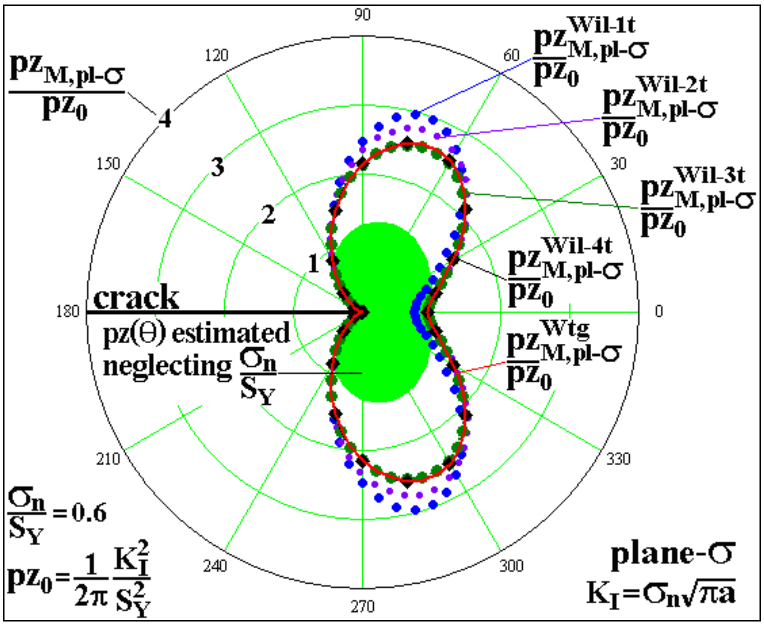


Figure 5: The Mises *pz(θ)* estimated for the Griffith plate loaded in mode I from the Williams series with only 3 terms visually reproduces reasonably well $pz_{Ing}(\theta) = pz_{Wtg}(\theta)$ both in *pl-σ* and in *pl-ε*.

But before doing so, it is important to remember that although the complete field generated e.g. from the Westergaard stress function is the correct LE solution for the Griffith plate, its truncation inside the plastic zone limits stresses, thus inevitably leads to underestimated *pz(θ)* frontiers. In a first approximation, such stresses can be limited by S_Y, neglecting strain-hardening effects inside *pz(θ)*, but such effects can be considered assuming an HRR-like stress-strain relation. However, due to space limitations only the ideal perfectly plastic behavior is discussed here.
Four alternative models for compensating the stress truncation inside the plastic zones augmenting them by forcing the plate to obey equilibrium conditions are considered following.

- Correction to compensate for the σ_y component truncation, as proposed by Rodriguez, Castro e Meggiolaro:(18)

$$pz_M^{Wtg+eq\sigma_y}(\theta) = \frac{\displaystyle\int_0^{pz_M^{Wtg}(\theta)} \{Re[Z(r,\theta)+y\,Im[Z'(r,\theta)]\}\,dr}{\sigma_y\left(pz_M^{Wtg}(\theta),\theta\right)} \tag{10}$$

This correction may be seen as a generalization of Irwin´s classical correction for the plastic zone along the crack direction $\theta = 0$, which is based on the equilibrium of net vertical forces that could not exist within the plastic zone because σ_y cannot surpass the yielding stress [3-4]. Besides the generalization to perform this correction along any θ-direction, the most important difference between equation (8) and Irwin´s receipt is that the former is based on the complete Westergaard stress function while the latter considers a stress field that is based solely on the SIF.

- Correction using a constant increment along each radius connecting the crack tip to the *pz(θ)* borderline, defined by its θ-direction, obtained from

$$pz_M^{Wtg+eqR}(\theta) = pz_M^{Wtg}(\theta) + CTE \tag{11}$$

where $CTE = pz_M^{Wtg+eqR}(\theta = 0) - pz_M^{Wtg}(\theta = 0)$. This constant has the same equilibrium rational as the previous correction, and it is based on the $\theta = 0$ direction. For other radial directions, the same length correction is adopted, inspired by the idea of the constant T-stress correction.

- Correction based on the Mises stress, obtained from
-

$$pz_M^{Wtg+eqM}(\theta) = \frac{\int_0^{pz_M^{Wtg}(\theta)} \sigma_{Mises}(r,\theta)\, dr}{S_Y} \tag{12}$$

Since the correction proposed by Rodriguez et al. $pz_M^{Wtg+eq\sigma y}(\theta)$ only presents the equilibrium rational for $\theta = 0$ in pure mode I and does not take into account the effect of the other stress components, this correction based on the Mises stress may be seen as a reasonable alternative, since it considers them and can be used for any type of loading.

- Correction based on the vertical traction component, obtained from
-

$$pz_M^{Wtg+eqTr}(\theta) = \frac{\int_0^{pz_M^{Wtg}(\theta)} t_y(r,\theta)\, dr}{t_y\left(pz_M^{Wtg}(\theta),\theta\right)} \tag{13}$$

where t_y is determined by

$$\begin{Bmatrix} t_x(r,\theta) \\ t_y(r,\theta) \end{Bmatrix} = \begin{bmatrix} \sigma_x(r,\theta) & \tau_{xy}(r,\theta) \\ \tau_{xy}(r,\theta) & \sigma_y(r,\theta) \end{bmatrix} \begin{Bmatrix} cos(\theta) \\ sin(\theta) \end{Bmatrix} \tag{14}$$

Again, this equilibrium correction to compensate for the LE stress field limitation inside the plastic zone has an exact equilibrium appeal only for $\theta = 0$. However, by considering the vertical traction component, the equilibrium may be seen as resulting from a free body diagram obtained by sectioning the model along any θ-direction, a more elegant way to treat this problem. Figures 6 and 7 compare the various equilibrium corrections described above, by showing the difference between their $pz(\theta)$ estimates for plane stress and plane strain. These figures also depict plastic zones obtained by truncated SIF, SIF plus T-stress, and complete LE stress fields, which do not obey equilibrium requirements. Note in particular that the K_I+T-stress $pz(\theta)$ can be significantly smaller than equilibrium-corrected ones.

ISSN 1516-392X

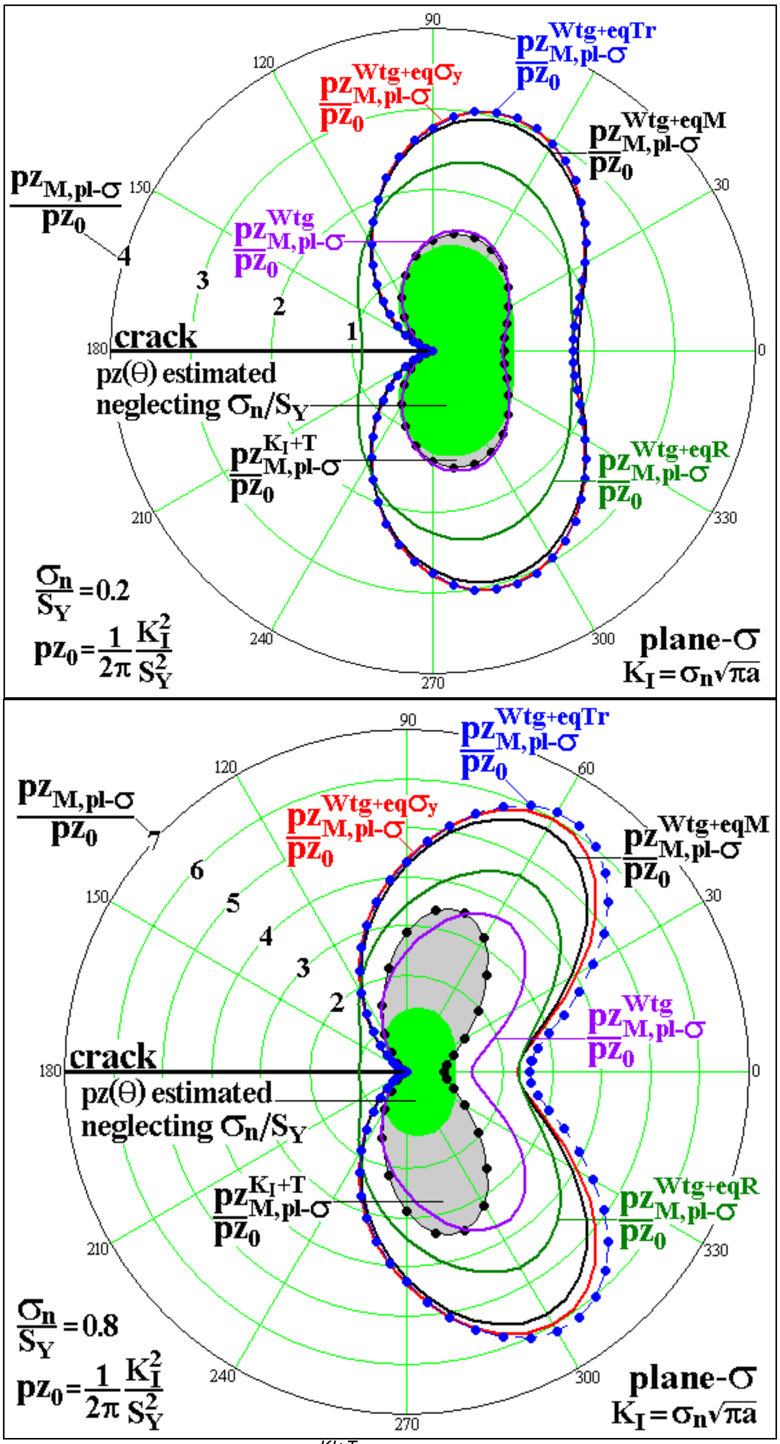


Figure 6: Equilibrium-corrected $pz(\theta)$ and $pz^{KI+T}(\theta)$ estimated for the Griffith plate loaded in mode I in $pl\text{-}\sigma$ for a low $\sigma_n/S_Y = 0.2$ and a high $\sigma_n/S_Y = 0.8$ ratios.

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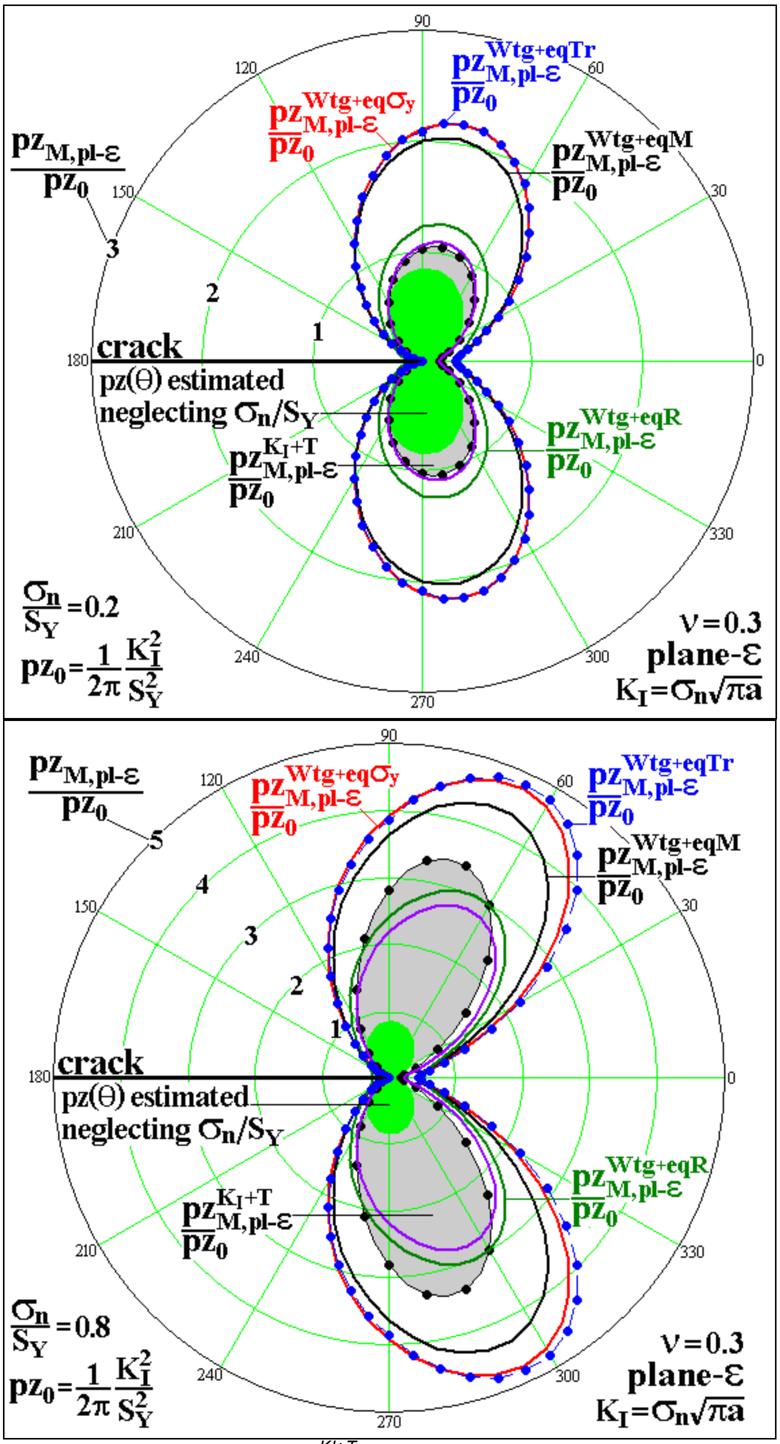

Figure 7: Equilibrium-corrected *pz(θ)* and $pz^{K_I+T}(\theta)$ estimated for the Griffith plate loaded in mode I in *pl-ε* for a low $\sigma_n/S_Y = 0.2$ and a high $\sigma_n/S_Y = 0.8$ ratios.

Figures 6 and 7 display *pz(θ)* borderlines estimated for $\sigma_n/S_Y = 0.2$ and $\sigma_n/S_Y = 0.8$ ratios, which correspond to yield safety factors $\phi_Y = 5$ and $\phi_Y = 1.25$, representative of maxima low and high loads used in typical structural applications. The equilibrium-corrected hypothesis based on σ_y, σ_M, and on the traction vector provide similar *pz(θ)* predictions, which are significantly larger than the K_I+T-stress one usually accepted as reliable *pz(θ)* estimates for analysis and design purposes. As the K_I+T-stress field neglects stress components considered by the exact LE solution for the Griffith plate, this suggest that for practical applications *pz(θ)* in generic cracked components should be estimated using equilibrium-corrected LE stress fields properly calculated using standard finite element procedures.[19]

6 CONCLUSIONS

The nominal stress to yield strength σ_n/S_Y ratio significantly affects both the size and shape of plastic zones ahead of crack tips estimated from LE stress fields, as demonstrated for Griffith's plate using 3 different ways to find its exact solution. This solution should be corrected to consider equilibrium requirements violated by the LE stress truncation inside the plastic zone, a task tackled by 4 different approximate but reasonable hypotheses. From these, the stress-based ones generate quite similar $pz(\theta)$ estimates. Such equilibrium-corrected $pz(\theta)$ are significantly larger than the estimates obtained from the plate SIF $K_I = \sigma_n \sqrt{(\pi a)}$ alone, or from the combination of its SIF+T-stress, particularly for the high σ_y/S_Y ratios used in modern structures.
As such estimates are based on an exact LE solution complemented by quite sensible equilibrium assumptions, they indicate that the traditional practice of assuming that T-stress can adequately correct SIF limitation for estimating $pz(\theta)$ may, and probably should be questioned. Moreover, they suggest that $pz(\theta)$ frontiers can be similarly estimated in cracked structural components using complete LE stress fields calculated by well-established finite element procedures, which should be then equilibrium-corrected to avoid underestimation due to stress truncation, possibly including strain-hardening effects for better precision. This fact has important practical consequences, as it can be used to seriously question the similitude principle, one milestone of the mechanical design against fracture, in many real life problems. In compensation, it may help to better predict the actual toughness of real structures, by comparing reliable estimates for their $pz(\theta)$ with those obtained for the standard test specimens used to measure their material toughness.

Acknowledgements

CNPq has provided research scholarships for the authors.

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