PREVISÃO DO LIMITE DE FADIGA DE PEÇAS ENTALHADAS PELO MODELO DO GRADIENTE DE TENSÃO E PELA TEORIA DA DISTÂNCIA CRÍTICA*

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Resumo
O modelo do Gradiente de Tensão (SG) e a Teoria da Distância Crítica (TCD) são usados para estimar o limite de fadiga de corpos de prova com entalhes alongados, e essas estimativas são testadas em C(T) modificados submetidos a tensões cíclicas de amplitude constante com uma razão de tensão R = 0.1. Os valores medidos são comparados com as previsões feitas por ambos os modelos. As previsões do modelo SG estiveram mais de acordo com os resultados experimentais. Também foi demonstrado que o modelo SG é quase equivalente à TCD no caso de entalhes com raios grandes, quando a TCD usa o método do ponto. Para o caso de entalhes afiados, os valores dos limites de fadiga previstos pela TCD são mais conservativos que os previstos pelo modelo SG.
Palavras-chave: Limite de fadiga; Modelo do gradiente de tensões (GT); Teoria da distância crítica (TCD).

FATIGUE LIMIT PREDICTION FOR ELONGATED NOTCHED SPECIMENS USING THE STRESS GRADIENT MODEL AND THE CRITICAL DISTANCE THEORY

Abstract
The Stress Gradient (SG) model and the Theory of the Critical Distance (TCD) are used to estimate the fatigue limit of elongated notched specimens, and such estimates are tested in modified C(T) specimens loaded under constant amplitude cyclic stresses and a stress ratio R=0.1. The measured values are compared with those predicted by both models. The SG model predictions agreed better with the experimental results. It is also demonstrated that the SG model and the TCD using the point method are almost equivalent for blunt notches, whereas for sharp notches the fatigue limit values predicted by TCD model are more conservative than those predicted by the SG model.
Keywords: Fatigue limit; Stress gradient (SG) model; Theory of critical distance (TCD).

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1 INTRODUCTION

Most failures in machinery and structural components can be attributed to fatigue damage processes. Such failures generally take place under the influence of cyclic loads whose peak values are considerably smaller than the safe loads estimated on the basis of static fracture analysis. Fatigue damage is characterized by the gradual initiation and/or propagation of a crack, which can eventually cause the failure of those structural components.

Fatigue limits of structural components can be estimated by many semi-empirical and theoretical methods that can be divided based on their approaches as total-life, critical-distance, and fracture-mechanics approaches. The SN and ε-N procedures use the total-life approach which does not recognize cracks. The classical critical-distance procedures are those based on the stress at a certain point ahead of the notch tip, initially proposed by Peterson Erro! Fonte de referência não encontrada., Neuber 2 and Heywood 3. Recent research, based on the average of the stress over a certain distance or in a certain representative volume of the material, has been developed by Taylor [4-5] with the so-called Theory of the Critical Distance (TCD), as explained in Section 1.1.

On the other hand, another philosophy to predict fatigue limits has been proposed based on linear elastic fracture mechanics (LEFM) concepts. Its main advantage is to recognize cracks and their effects on the fatigue limit, modeling the actual fatigue limit more realistically because there are no defect-free materials in practice. The pioneer model based on this methodology was proposed by El Haddad, Toper and Smith (ETS) 4, and it has been recently extended to consider notch effects [7-9] in the so-called Stress Gradient (SG) model briefly described in Section 1.2.

1.1 TCD model

In TCD model Taylor assumes a characteristic material length parameter, called the critical distance *L* (Equation 1), that includes mechanical properties such as ΔK_0 and ΔS_{L0} (the long crack propagation threshold and the smooth specimen fatigue limit for a stress ratio $R = \sigma_{min}/\sigma_{max} = 0$, where σ_{min} and σ_{max} are the minimum and maximum values of the stress in the cycle). The TCD group includes the Point Method (PM), the Line Method (LM), the Area Method (AM), and the Volume Method (VM), which is the most general one. To make predictions, the TCD model requires that the elastic stress range (in the loading direction) to be known as a function of its distance *x* ahead of the notch tip, $\Delta\sigma(x)$.

$$L = \frac{1}{\pi}\left(\frac{\Delta K_0}{\Delta S_{L0}}\right)^2 \qquad (1)$$

1.1.1 The Point Method (PM)

The PM is the simplest form of the TCD. In this approach, the criterion for crack propagation (fatigue limit) is that the local stress at a distance $x=L/2$ ahead of the notch tip equals to the smooth specimen fatigue limit ΔS_{L0}. It can be expressed mathematically by Equation 2.

$$\Delta\sigma\left(L/2\right) = \Delta S_{L0} \qquad (2)$$

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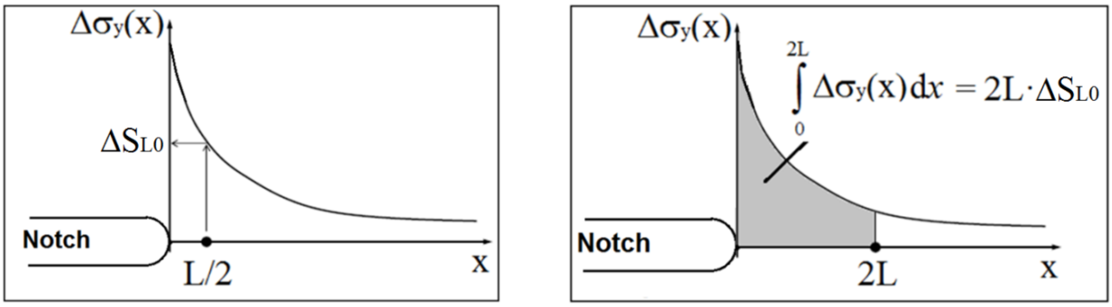
The PM applied on a notch is illustrated in Figure 1a. At the fatigue limit the stress range at $x = L/2$ is the smooth specimen fatigue limit ΔS_{L0}.

1.1.2 The Line Method (LM)

The LM method uses an average stress over a distance $x = 2L$ from the notch tip rather than a stress at particular point, like in the PM method. For the fatigue limit determination it is required that such average stress equals to the smooth specimen fatigue limit of ΔS_{L0}. Mathematically, it can be expressed by Equation 3.

$$\frac{1}{2L}\int_{0}^{2L}\Delta\sigma(x)\,dx = \Delta S_{L0} \quad (3)$$

The LM method applied on a notch is shown in Figure 1b. At the fatigue limit the average stress amplitude over $x = [0, 2L]$ is the smooth specimen fatigue limit ΔS_{L0}.



(a) (b)

Figure 1. a) PM and b) LM methods applied on the elastic stress field ahead of a notch tip [6].

1.1.3 The Area (AM) and Volume (VM) Methods

The AM involves an average stress over some area in the vicinity of the notch tip, whereas the VM makes use of a volume average. Considering a semicircular area, or a hemispherical volume in the VM, centered on the notch root, Bellet et al. [10] showed that the radius of the semicircular area is $1.32{\cdot}L$ and that of the hemispherical volume is $1.54{\cdot}L$. However, the PM and the LM methods are more used because they are easier to apply.

1.2 The SG model

This model predicts the behavior of short cracks that depart from notch tips by fatigue (or EAC). It considers that although such cracks can be easily generated at sharp notches, which introduce high stress concentration effects at their tips, due to the high stress gradient acting around these tips, the short cracks can also stop to grow by fatigue after having propagated through a small distance, thereby becoming non-propagating cracks that can be tolerated in service if $\Delta S_{L0}/K_t \leq \Delta\sigma_n \leq \Delta S_{L0}/K_f$ [11], where $\Delta\sigma_n$ is the nominal stress range, and K_t and K_f are the linear stress concentration factor and the fatigue stress concentration factor, respectively.

Short cracks behave differently from long cracks, see the Kitagawa-Takahashi plot trend [12] in Figure 2. Their bi-linear behavior was modeled by the ETS model using the characteristic crack size a_0, which is estimated from ΔS_{L0} and ΔK_0. Their correct asymptotic behaviours reproduce both the fatigue limit and the FCG threshold, i.e., $\Delta\sigma(a \leq a_0) = \Delta S_{L0}$ for short cracks, and $\Delta K_0(a \geq a_0) = \Delta K_0$ for long ones, is obtained using a modified stress intensity factor (SIF) range ΔK_I to describe the fatigue propagation of both short or long cracks, as shown in Equation 4.

$$\Delta K_I = \Delta\sigma\sqrt{\pi(a+a_0)} \text{ , where } \quad a_0 = \left(\frac{1}{\pi}\right)\left(\frac{\Delta K_0}{\Delta S_{L0}}\right)^2 \qquad (4)$$

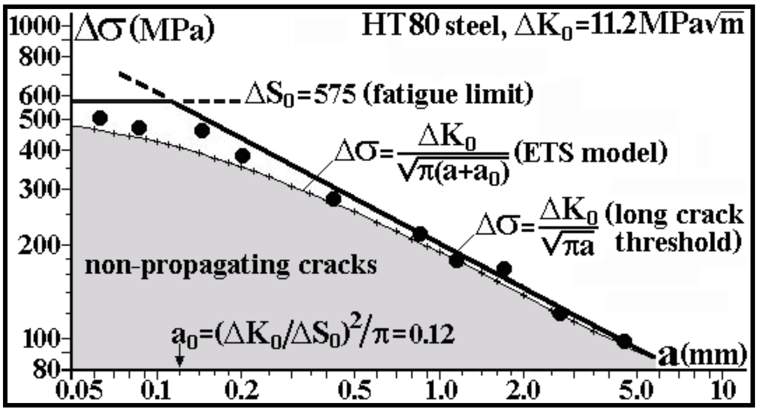


Figure 2. Kitagawa-Takahashi plot describing the fatigue propagation of short and long cracks under pulsating loads (R = 0) in a HT80 steel with ΔK_0 = 11.2[MPa√m] and ΔS_0 = 575[MPa] [13].

Equation 4 can be generalized to be applied to other geometries:

$$\Delta K_I(a) = \alpha \cdot f(a,\rho) \cdot \Delta\sigma_n\sqrt{\pi(a+a_0)} \text{ , where } \quad a_0 = \left(\frac{1}{\pi}\right)\left(\frac{\Delta K_0}{\alpha \cdot \Delta S_{L0}}\right)^2 \qquad (5)$$

where $f(a,\rho)$ describes the stress gradient ahead of the notch tip, which tends to K_t as the crack length $a \to 0$, and α encompasses all the remaining terms, such as the free surface correction.

Notice that the so-called critical distance L is similar to ETS's characteristic crack size a_0, except that it does not include the free surface correction α. Hence, the critical distance can be also calculated as $L = a_0 \cdot \alpha^2$. From ETS model, the threshold SIF expression can be modified to become a function of the crack length a, so to include both the short and the long crack behavior, resulting in Equation 6 [7].

$$\Delta K_{th}(a) = \Delta K_0 \left[1 + \left(\frac{a_0}{a}\right)^{\gamma/2}\right]^{-1/\gamma} \qquad (6)$$

where $\Delta K_{th}(a)$ is the FCG threshold as a function of the crack size a, and γ is the adjustable Bazant's [14] parameter that can be used to better fit experimental data (for most structural alloys, $1.5 < \gamma < 8$ [7].)

Hence, there is no crack propagation if $\Delta K_I(a) < \Delta K_{th}(a)$. At the fatigue limit it is required that $\Delta K_I(a) = \Delta K_{th}(a)$ and following the analysis detailed in [7], it can be demonstrated that the fatigue stress concentration factor $K_f = \Delta S_{L0}/\Delta\sigma_n$, and consequently, the fatigue limit a notched component can be predicted by solving Equation 7.

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$$\begin{cases} f\left(a_{\max},\rho\right)=g\left(a_{\max},K_f,\Delta K_0/\Delta S_{L0},\gamma\right) \\ \dfrac{\partial}{\partial a} f\left(a_{\max},\rho\right)=\dfrac{\partial}{\partial a} g\left(a_{\max},K_f,\Delta K_0/\Delta S_{L0},\gamma\right) \end{cases} \quad (7)$$

where a_{max} is the largest non-propagating flaw that can arise from fatigue alone, limiting the condition of propagating and non-propagating cracks. It is worth noting that g is a dimensionless function defined by Equation 8.

$$g=\left(\frac{\Delta K_0}{\Delta S_{L0}}\right)\cdot\left(\frac{\Delta S_{L0}}{\Delta\sigma_n}\right)\cdot\left[\left(\alpha\cdot\sqrt{\pi a}\right)^{\gamma}+\left(\frac{\Delta K_0}{\Delta S_{L0}}\right)^{\gamma}\right]^{-1/\gamma} \quad (8)$$

It is important to point out that the traditional fatigue analysis of notched specimens by TCD model is done using an empirical formula, which lacks physical meaning. Conversely, the key assumption of the SG model is that the fatigue process is indeed a crack growth process that can be predicted using the stress intensity factor as a fracture mechanics-based driving force. Therefore, the aim of this work is to estimate the fatigue limit in an elongated notched specimen subjected to a constant amplitude cyclic stress using these two approaches and compare these values with the experimentally measured fatigue limit.

2 MATERIALS AND EXPERIMENTAL PROCEDURE

Modified C(T) specimens with an elongated notch machined from a 76.2[mm] x 12.7[mm] flat bar of SAE 1020 steel were used to verify the fatigue limit predictions. The chemical composition of this steel given by the supplier is presented in Table 1, while the basic mechanical properties are shown in Table 2. These properties were measured in a 100[kN] INSTRON servocontrolled testing machine at a crosshead speed of 0.9[mm/min] according with ASTM E 8M-13a standard.

Table 1. The chemical composition of the tested SAE 1020 steel

Element	**Mn**	**Si**	**C**	**Cr**	**Cu**	**Ni**
wt%	0.36	0.24	0.2	0.19	0.1	0.04

Table 2. The basic mechanical properties of the tested SAE 1020 steel

S_Y [MPa]	S_U [MPa]	E [GPa]
317	490	197

The specimens were tested under fixed range force-controlled, at a frequency of 90 [Hz]. Typical fatigue limits are specified at lives of 10^6-10^7 cycles for steels, thus, in this work it was used a live of $3{\cdot}10^6$ cycles for assessing the fatigue limit of the modified C(T) specimens. An accelerated fatigue test involving step loading was carried out following procedures proposed in [15].
The smooth specimen fatigue limit at R = 0.1, $\Delta S_{L0.1}$ = 412.2 MPa, was measured by the conventional up-and-down method [16] testing 12 specimens machined according to ASTM E606M-12 standard. All the tests were performed under uniaxial constant amplitude stress-control loads with a stress ratio R= 0.1 and at 57[Hz] frequency using the same testing machine described above. The specified criterion to failure was $2{\cdot}10^6$ cycles.

The long crack propagation threshold $\Delta K_{0.1}$=10 MPa√m was measured according to ASTM E647 procedures. Standard C(T) specimens were tested under *K*-decreasing and *K*-increasing methods with a stress ratio R = 0.1 using the same testing machine described above. From measured $\Delta S_{L0.1}$ and $\Delta K_{0.1}$, the characteristic crack size was computed using Equation 5 ($a_{0.1}$ = 149 μm).
The width and thickness of the modified C(T) specimens, *W* = 60 mm and *B* = 9 mm, respectively, are shown in Figure 3. Four combinations of notch depth to tip radius *b/ρ* were considered and are shown in Table 3. The smallest value of *ρ* was machined by electro-erosion and the others were previously undersize drilled and the excess material was removed using reamers to attain the final dimensions with a superior finish.

Table 3. Notch depth to tip radius combination for the modified C(T) specimen.

Notch depth, b [mm]	***Notch tip radius, ρ*** [mm]	***b/ρ***	***b/W***
15	0.15	100	0.25
20	1	20	0.33
23	0.75	30.7	0.38
25	0.62	40.2	0.42

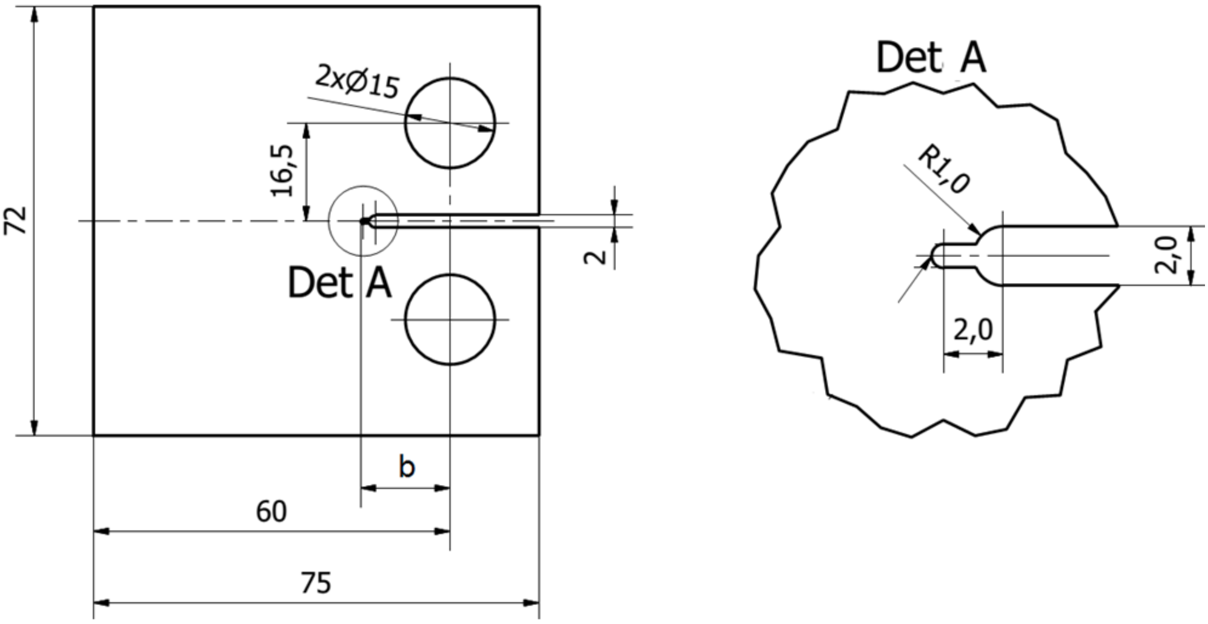

Figure 3. Dimensions of the modified C(T) specimen used for fatigue limit assessing.

3 RESULTS AND DISCUSSIONS

Following Frost's statement, the difference between K_t and K_f defines the generation of non-propagating short cracks. Numerical results for K_f predicted by SG model and TCD using PM method are plotted in Figure 4 as a function of the notch tip radius ρ, for *b*=15 mm. Moreover, Figure 4 shows K_t values computed by the Creager-Paris approach [17] and how them tend to K_f as ρ increases. Therefore, for a modified C(T) of SAE 1020 steel, notches with tip radii ρ ≤ 1.0 mm will be able to generate non-propagating cracks. For this reason, notch tip radii detailed in Table 3 were chosen in this work.

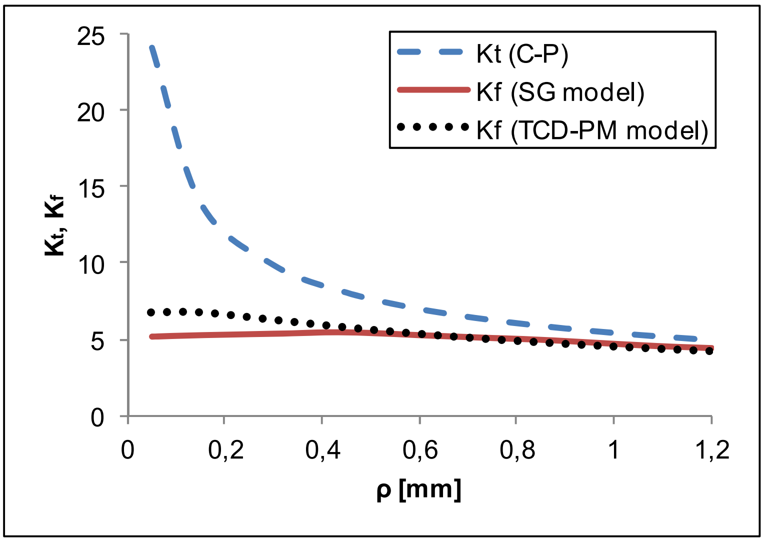

Figure 4. Comparison of predicted K_f from the SG and TCD models with K_t from Creager-Paris approach as a function of ρ, for a C(T) modified specimen with b = 15 mm.

The values of K_f predicted by SG and TCD-PM models as well as the experimental K_f values are presented in Table 4 for the four combinations of *b/ρ*. Note that K_f values estimated by the SG model agree better with the experimental ones. However the TCD-PM model provides more conservative predictions of K_f. The models predictions differ more for high K_t values and are almost coincident for lower K_t values, see Figure 5. This means that SG and TCD-PM models predictions almost overlap for blunt notches. It is relevant to mention that the criterion to consider fatigue failure was established when the crack length (a) was higher than 1.5 mm (~$10 \cdot a_{0.1}$), since that value for SAE 1020 steel is considered a long crack.

Table 4. Notch depth to tip radius combination for the modified C(T) specimen.

b [mm]	***ρ*** [mm]	K_t (C-P)	K_f (TCD model)	K_f (SG model)	K_f (Experim.)
15	0.15	13.89	6.72	5.3	---
20	1.00	5.10	4.57	4.45	3.57
23	0.75	5.61	5.02	4.6	3.54
25	0.62	5.94	5.31	4.6	3.81

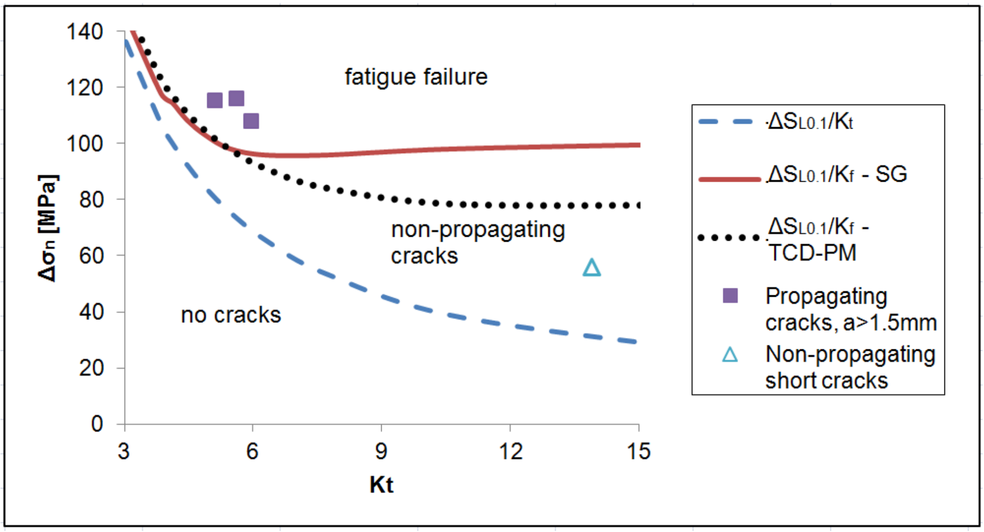

Figure 5. Comparison of experimental and predicted data for the C(T) modified specimen. The superior limit of the region of non-propagating cracks are defined by the corresponding curve of each model.

During the experimental assessment of the fatigue limit, a short crack (a ≈ 75 μm) was detected on the surface of specimen with b = 15 mm and ρ = 0.15 mm, with the aid of an optical microscope (ZEISS, model *Axioplan* 2) and this crack did not propagate after $3.1{\cdot}10^6$ cycles. The initial and the ending point of this short crack are shown in Figure 6. This point lies on the non-propagating cracks region showed in Figure 5. The largest non-propagating crack a_{max} determined using the SG model after solving the Equation 7 was 720 μm, which corroborates that the detected crack is indeed a non-propagating short crack.

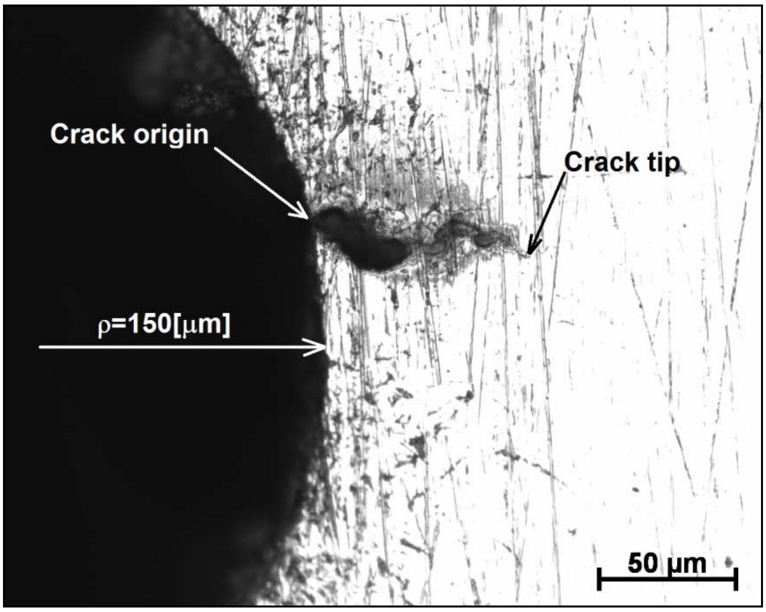

Figure 6. Short non-propagating crack with a ≈ 75 μm at notch tip of C(T) specimen with b = 15 mm and ρ = 0.15 mm, under $\Delta\sigma_n$ = 56 MPa, depicted after $3.1{\cdot}10^6$ cycles.

4 CONCLUSIONS

The predictions of the SG model agreed better with the fatigue limit experimentally measured on SAE 1020 steel C(T) specimens modified to include a notch. It was also demonstrated that SG model and TCD model using the point method are almost equivalent for blunt notches, whereas for sharp notches the fatigue limit values predicted by TCD model are more conservative than those predicted by the SG model.

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