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REDUCED ORDER MODELING OF COMPOSITE LAMINATES THROUGH SOLID-SHELL COUPLING[1]

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Abstract

Composite laminates display a complex behavior due to their microstructure, with a through-thickness variation of the displacement and stress fields dependent on the fiber orientation in each layer. Aiming to develop reduced-order numerical models mimicking the real response of composite structures, we investigate the capability and the accuracy of finite element analyses coupling layered shell and solid kinematics. This study represents the first step of a work with the goal of accurately matching stress evolution in regions close to possible impact locations, where delamination is expected to take place, with reduced computational costs. Close to such locations a three-dimensional space modeling is adopted, whereas in the remainder of the structure a less demanding shell modeling is chosen. To test the coupled approach, results of numerical simulations are presented for a quasi-statically loaded cross-ply orthotropic plate, either simply supported or fully clamped along its boundary.

Keywords: Composite laminates; Finite element simulations; Reduced order modeling; Solid-shell coupling.

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1 INTRODUCTION

In this work we focus on dissipative mechanisms, micromechanically linked to damage processes located inside the resin-enriched zones between adjacent layers, i.e. on delamination.(1-4) Even if numerical simulations of delamination events are not reported, we frame the problem within the so-called meso-mechanical approach. We account for the small thickness of the resin-enriched regions to develop simplified computational models resting on a lumping of the damage-induced interlaminar decohesion onto zero-thickness surfaces.(5-8) Such lumping procedure is based on overall constitutive laws for each resin region, defined in terms of boundary tractions vs displacement jumps; the tractions are those acting on the top and bottom surfaces of any representative volume of the interlaminar region, while the displacement jumps are measured as variations between the two aforementioned opposite surfaces.

To model interlaminar damage processes, the through-thickness variation of the displacement field in displacement-based finite element (FE) simulations is required, through e.g. three-dimensional (3D) space discretizations. A fully solid discretization of the structural component would result in a heavy computational burden for real-life cases. The other way around, a shell-like two-dimensional (2D) modeling proves sufficient for regions not exposed to delamination events. Here we adopt a hybrid approach, able to ensure accuracy with limited computational costs, by coupling a solid kinematics to a shell one away from the delaminating zones. A similar approach was already considered in Reinoso et al.(9) to model the damage evolution in the skin-stringer joint of a complex composite sample for aeronautical applications, see also Corigliano, Maier and Mariani.(10) In these and similar works, accuracy and efficiency assessments were not reported.

Focusing on coupled modeling for thin plates undergoing bending deformations induced by a distributed load, we provide an analysis of the approach in terms of plate deflection, elastic energy stored in the laminate, through-thickness variation of the stress fields, and speedup with respect to the fully solid models. Whenever possible, results are compared to the outcomes of the 3D Reddy-Pagano approach, and first-order shear deformation theory (FSDT).

In what follows, we highlight some computational features of the coupled procedure in Section 2. In Section 3, results are discussed for a $[90/0]_{6s}$ composite plate, either simply supported or fully clamped along its boundary. Eventually, some concluding remarks are gathered in Section 4.

2 SOLID-SHELL COUPLING AND COMPUTATIONAL ISSUES

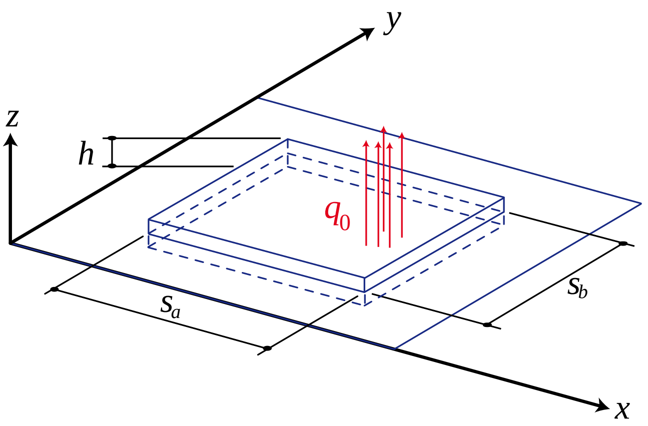

Figure 1. Plate bending problem, coupled solid-shell modeling: geometry and notation.

Let us consider a flat rectangular plate, as depicted in Figure 1. The plate is either simply supported or fully clamped along its boundary, and is quasi-statically loaded over its top surface by a uniform pressure q_0, which is positive if pointing outward (solution can be obviously generalized to the non-uniform load case(11)). An orthonormal reference frame is introduced, with axes x and y located on the mid-plane and axis z pointing in the same direction of the pressure.

We model with a full 3D kinematics the central portion of the plate, whose side lengths are s_a and s_b, see Figure 1; the remainder of the plate is instead modeled with a 2D shell-like kinematics. Accordingly, the former region is discretized with 3D brick elements, whereas the latter region is discretized with 2D shell elements.

Numerical simulations here collected have been run with the FE code Abaqus.(12) In what follows we discuss two topics that affect the accuracy of the solutions provided by the hybrid 2D-3D model: the shell kinematics, to avoid shear locking; the coupling between the solid and shell structural regions.

As far as the shell kinematics is concerned, restricting the analysis to the small displacement regime, the 4-node (reduced-integration) S4R element has been adopted. The through-thickness variation of the strain and stress fields featured by such element is compliant with the FSDT;(13,14) this kinematics implies that segments normal to the mid-plane retain their straight shape when deformed. Even if shear correction factors are adjusted in case of anisotropic materials, and an inhomogeneous material response can be allowed for along the thickness direction, the transverse shear deformations are assumed constant throughout the whole thickness. This represents an approximation of the real composite behavior, which is instead characterized by a zigzagging of the fields.(15) The relevant reduced-integration approach, which is developed on the basis of an assumed strain one via a Hu-Washizu variational principle, coupled to a stabilization to avoid zero-energy deformation patterns,(16,17) guarantees the avoidance of shear locking in case of thin plates.

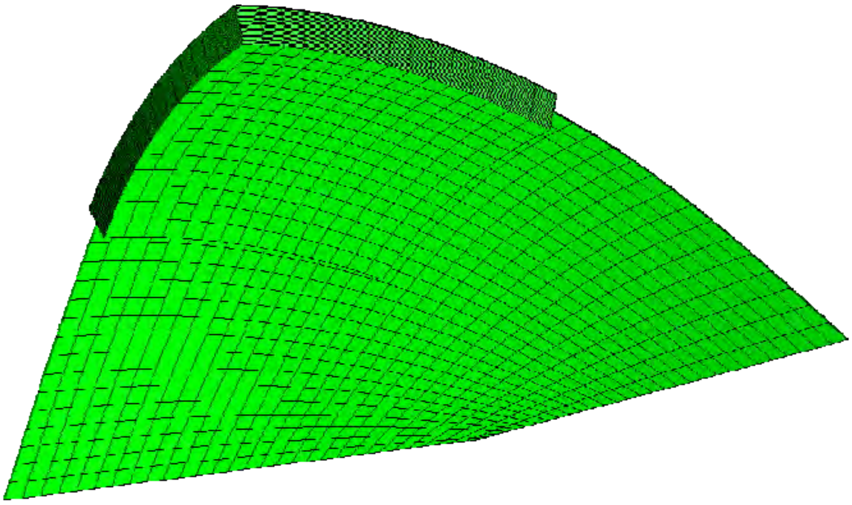

Figure 2. Example of the deformed configuration of bent composite plates, modeled with the hybrid solid-shell approach (plate deflection has been magnified to highlight the quality of coupling along the border between the solid and shell regions).

The coupling between solid and shell regions of the plate is obtained through a surface-based interaction. This technique allows matching the displacements of nodes on the border of the solid region to the displacements and rotations of nodes on the border of the shell region, so as not only displacement jumps but also rotation jumps (i.e. kinks) are locally prevented, see Figure 2. Because of the S4R shell kinematics mentioned above, which cannot model the zigzagging of the fields along the through-thickness direction, the coupling introduces a local perturbation of the solution. To assess the effects of such perturbations on the modeled structural response, in the next Section the size ratios $\alpha = s_a/a$ and $\beta = s_b/b$ (see Figure 1) are varied in the range $0.25 \leq \alpha, \beta \leq 0.75$.

3 RESULTS: $[90/0]_{6s}$ LAMINATE

A symmetric cross-ply $[90/0]_{6s}$ laminate is considered; the side lengths of the composite plate are $a = b = 127$ mm, while the thickness is $h = 3.36$ mm. A uniform distributed load $q_0 = 2$ MPa is adopted. The elastic properties of each transversely isotropic layer, in a local x_1, x_2, x_3 reference frame with x_1 aligned with the fiber axis, are$^{(11)}$: $E_{11} = 144{,}000$ MPa, $E_{22} = E_{33} = 9690$ MPa, $G_{12} = 55{,}385$ MPa, $G_{13} = G_{23} = 5760$ MPa, $\nu_{13} = \nu_{23} = 0.3$. In what precedes, E is the Young's modulus, G the shear modulus, and ν the Poisson's ratio.

A preliminary investigation of the effects of meshing on the results has been carried out; what turned out is that the stacking sequence, and the relevant zigzagging of the in-plane strain and stress fields require a meshing with more than one element across the thickness of each layer to approach the FSDT solution. Anyhow, results are here presented in the case of one element discretizing each layer in the through-thickness direction, so as to assess the accuracy of the numerical solutions when minimal meshing and, therefore, minimal computing time are requested.

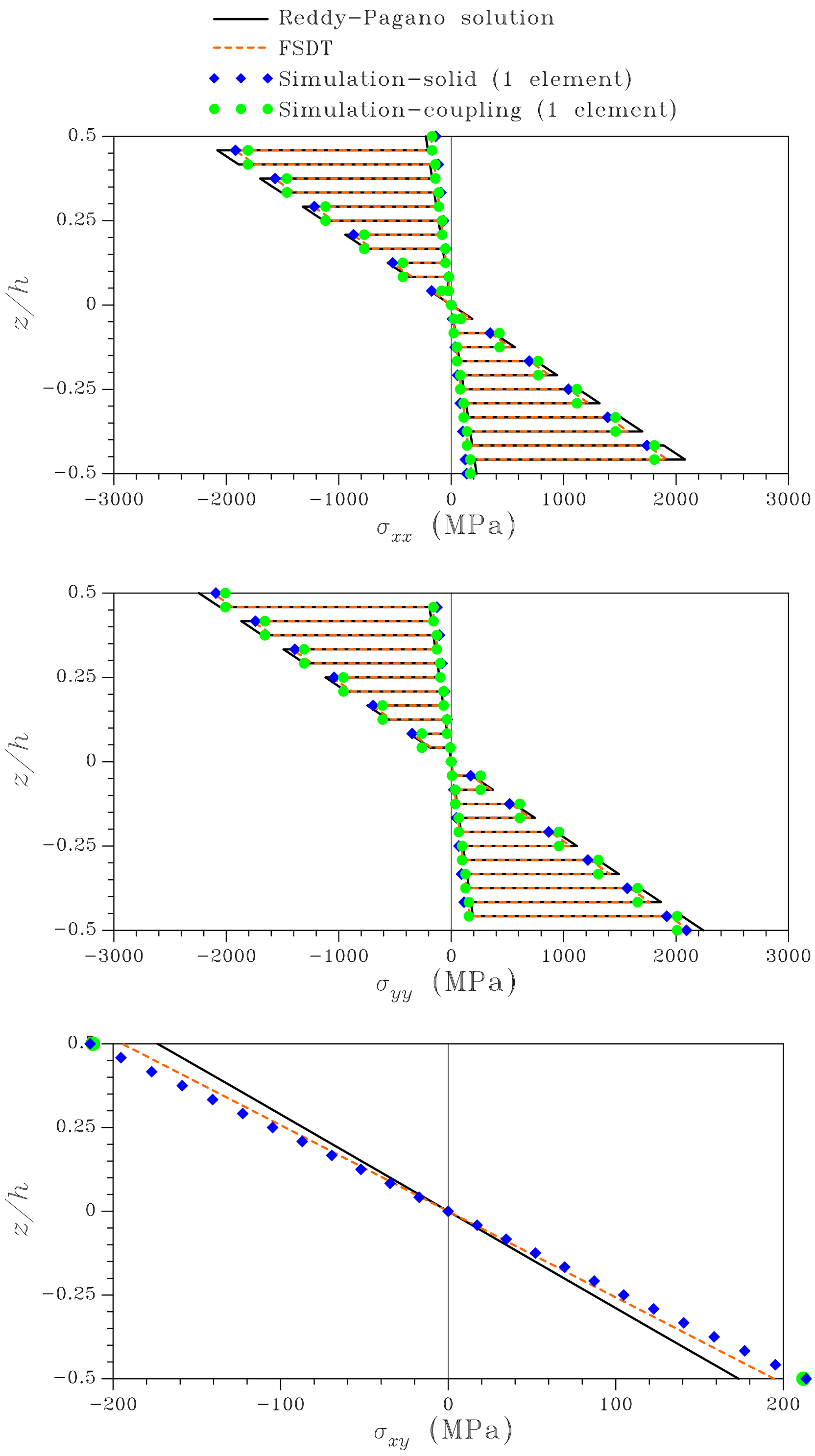

Figure 3. Simply supported laminate. Comparison among Reddy-Pagano and FSDT solutions and results of simulations, in terms of: (top) σ_{xx} and (middle) σ_{yy} at plate center; (bottom) σ_{xy} at plate corner.

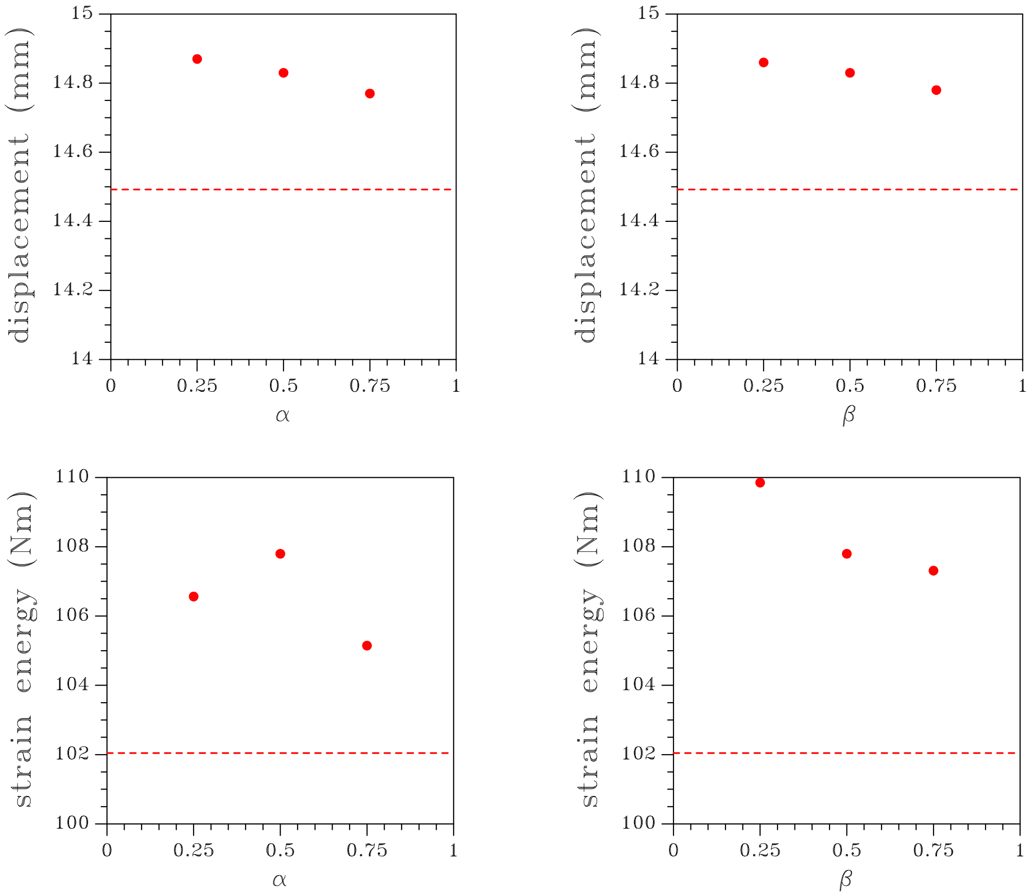

Figure 4. Simply supported laminate. Effect of coupling coefficients (left column) α (with $\beta = 0.5$), and (right column) β (with $\alpha = 0.5$) on: (top) central plate deflection; (bottom) stored strain energy.

As far as the simply supported case is concerned, results are reported in Figure 3 in terms of through-thickness variation of the stress field. While the fully solid discretization provides results almost perfectly matching the FSDT ones, at least at the plate center, the solid-shell coupling introduces a perturbation leading to reduced stress amplitudes.

Results in terms of central deflection and stored strain energy are shown in Figure 4, to understand the effect of modeling parameters α and β. The strain energy stored in the plate is here adopted to implicitly assess the in-plane size of the region affected by the distortion in the stress and strain fields due to the coupling of the solid and shell kinematics, keeping in mind that the zigzagging of the fields is not allowed for in the computational models. In the investigated intervals, it is shown that the impact of α and β is marginal: the maximum discrepancy with respect to a reference overkill simulation (dashed lines in the graphs) amounts to 2.6% as for the central deflection, and 7.6% as for the overall strain energy.

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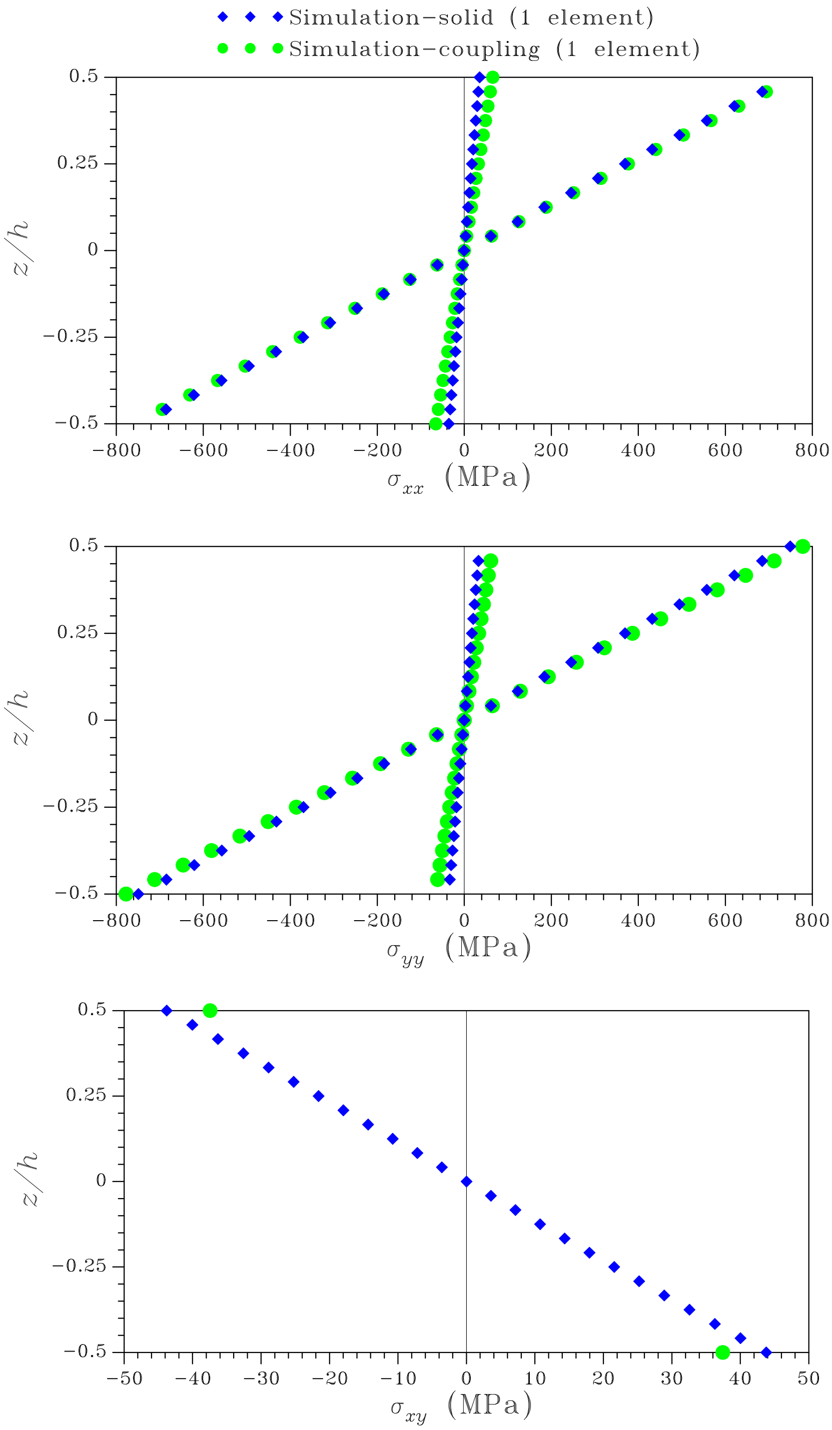

Figure 5. Clamped laminate. Comparison between results of simulations, either allowing for solid-shell coupling or adopting a uniform 3D space discretization, in terms of: (top) σ_{xx} and (middle) σ_{yy} at plate center; (bottom) σ_{xy} at plate corner.

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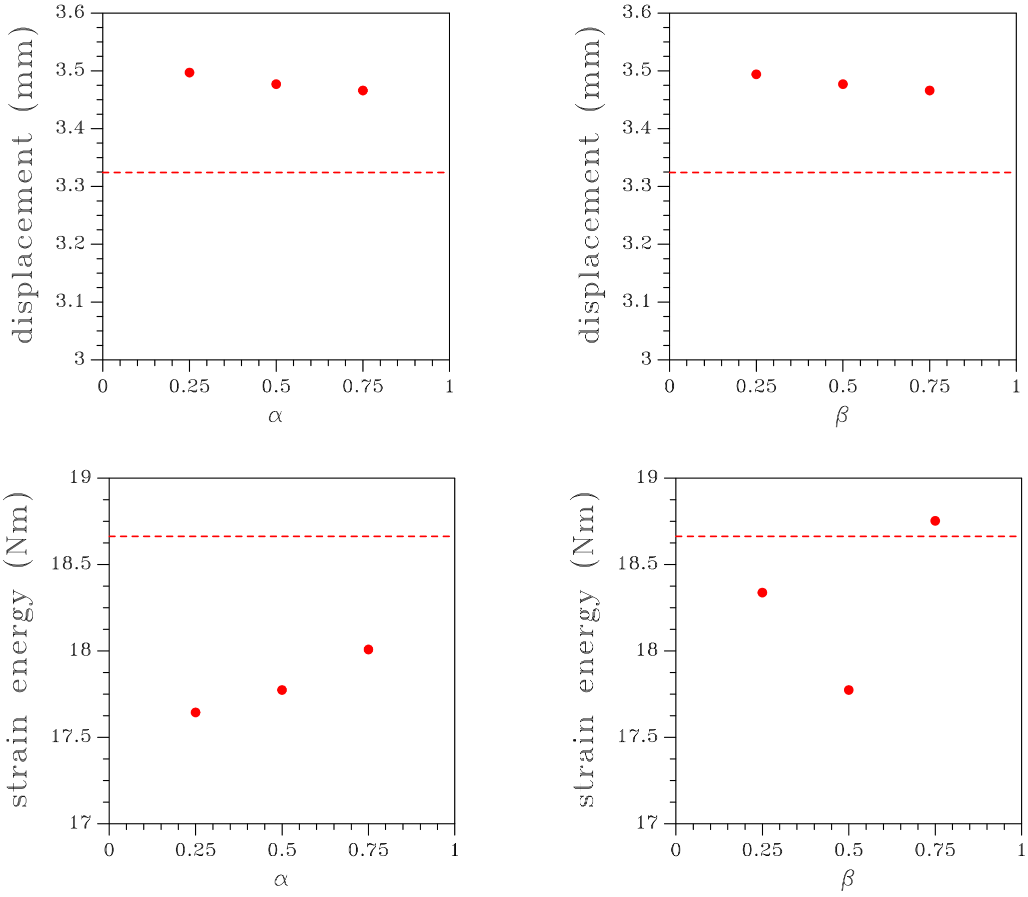

Figure 6. Clamped laminate. Effect of coupling coefficients (left column) α (with $\beta = 0.5$), and (right column) β (with $\alpha = 0.5$) on: (top) central plate deflection; (bottom) stored strain energy.

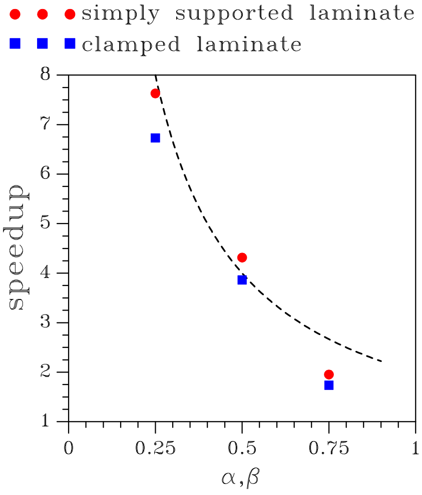

Figure 7. Effect of coupling coefficients α (with $\beta = 0.5$) or β (with $\alpha = 0.5$) on the speedup.

If the laminate is assumed to be fully clamped along its boundary, a comparison is reported in Figure 5, in terms of variation of the stress field along z, between the fully solid and the coupled solid-shell models featuring the same through-thickness discretization (namely, the minimal one with a single element across each lamina). In agreement with the simply supported case, the coupled model provides a slightly higher estimation of the stress level at the center of the plate, and an underestimation at the plate corner.

Figure 6 shows the results in terms of central deflection and stored strain energy, compared to the reference model. It can be seen that, even with a fully clamped boundary, the α and β ratios do not affect much the solution; at most, a discrepancy of 5.4% with respect to the fine mesh solution is reported.

To understand the computational gain achieved through the hybrid kinematics, which looks necessary to motivate the adoption of a complicated modeling of the composite plates, outcomes are gathered in Figure 7 in terms of speedup as a function of α or β, for both the considered boundary conditions. The speedup is measured as the ratio between the CPU time of the fully solid analysis, and the CPU time of the hybrid one featuring the stated values of α (when $\beta = 0.5$) or β (when $\alpha = 0.5$). The same in-plane discretization has been obviously adopted in the two aforementioned solutions; in the hybrid analyses, the same through-thickness discretization of the fully solid simulation has been adopted in the central plate portion. As a term of assessment, the dashed line in the plot reports how the inverse of the relative in-plane size of the solid region scales with α and β, i.e. $\left(\frac{s_a s_b}{ab}\right)^{-1} = (\alpha\beta)^{-1}$; Figure 7 shows that such scaling term can be roughly considered as a quite tight bound on speedup. The results here above have been obtained by running Abaqus on a laptop with Windows 7-64 bit as operating system, an Intel Core I7-2620 M @ 2.70 GHz as CPU, and 8 Gb of RAM.

4 CONCLUSION

In this work, a numerical investigation to assess the accuracy and efficiency of coupled solid-shell modeling for composite laminates has been provided.

Quasi-static or low-velocity impact loadings may lead to delamination processes, located along the interlaminar regions; hence, shell elements cannot be used in finite element modeling, as they do not account for jumps in the displacement field across the thickness. To capture the post-impact response of composite structures, a full 3D kinematics for the plate region surrounding the impact location looks necessary; in the remainder of the structure, where delamination is not expected to occur, a shell kinematics can instead prove sufficient.

In case of thin laminated plates, we have accordingly approached the problem by coupling solid and shell elements. For a $[90/0]_{6s}$ composite plate under a uniform distributed load, it has been shown that the through-thickness variation of the stress field is matched by the hybrid modeling with a good level of accuracy: a discrepancy well below 10%, with respect to reference numerical solutions, has been reported. It has been also shown that the speedup (computed as the ratio between the CPU time required to run the 3D solid analysis and the CPU time required to run the hybrid models) basically scales with the ratio between the volume of the whole composite and the volume of the 3D modeled region. In case of real-life composite structures, such approach is therefore expected to provide an excellent reduction of the computational costs.

A minimal computational burden can be obtained if the 3D region is assumed large enough to enclose all the possible delaminating regions; since the extent of delamination cannot be known a-priori, being dependent on the loading conditions, a mesh updating procedure needs to be designed so as the kinematics is locally switched from a 2D shell one to a 3D solid one every time a critical stress threshold is approached throughout the laminate thickness.

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