

AN ANALYSIS OF TEMPER MILLS IN HEAVY GAUGE LEVELING / CUT-TO-LENGTH LINES ¹

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Abstract

Heavy gauge flat plate formed from coils of hot rolled low carbon and mild steels is formed by powered roller levelers. The leveler adjusts the strain pattern by progressively counter-bending the strip over a series of rolls. The bending forms tensile stresses (non-contacting surface) and compressive stresses (surface in contact with the roll). The finite depth of yielding stress penetration leaves a non-yielded interior that retains the stress pattern of the incoming coil (residual). A flat plate is produced by balancing the plate's adjacent, coupled interior stresses (including residuals) so the composite stress pattern falls below the buckling threshold. Cutting the plate separates the coupled interior stresses leading to the pieces deflecting out of plane to find a new equilibrium. The nature of the incoming transverse strain pattern may cause some regions of the plate width to not experience sufficient yielding stress leading to Lüder Bands in low carbon steels. This paper examines the performance improvements of placing an in-line temper mill upstream of the leveler. The temper mill induces a full depth plastic deformation that leaves no residual stress pattern from the incoming coil, and yields the material well beyond the Lüder Band region. The temper mill can prepare the shape of the strip fed to the leveler, optimizing the leveler's bending adjustments and tracking.

Keywords: Temper mill; Heavy gauge strip; Leveling; Cut-to-length.

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1 INTRODUCTION

Heavy gauge, flat plate formed from coils of hot rolled low carbon and mild steels is a popular and well understood product. A traditional process flow is shown in Figure 1.

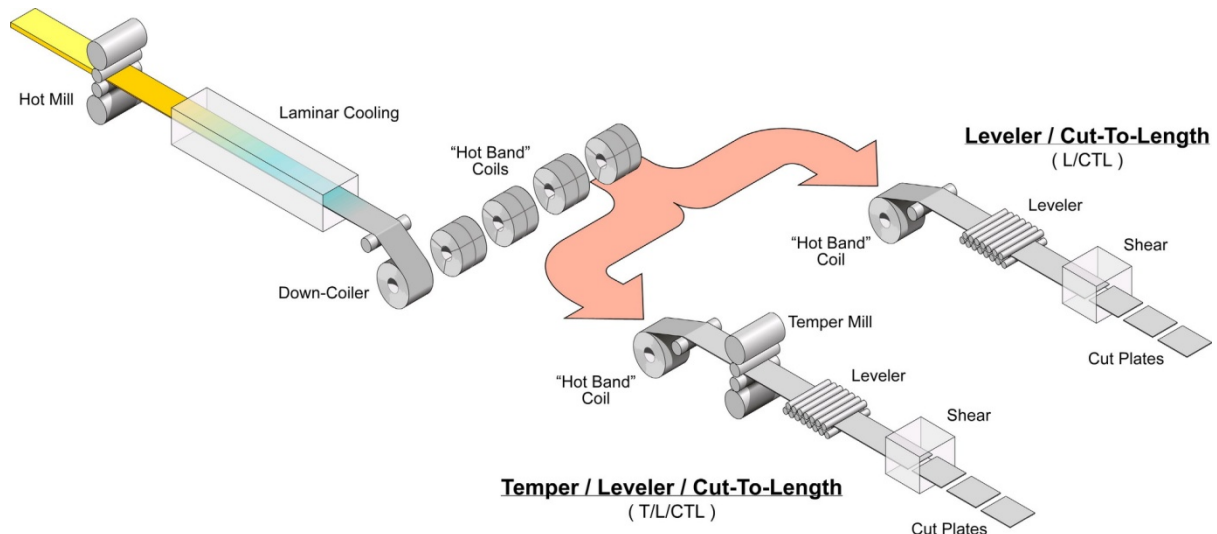


Figure 1 – Illustration of a typical “Hot Band” coil and “Cut-To-Length” process for producing heavy gauge flat plate.

When considering flat plate products, the coil finishing processing lines fall into two(2) primary forms:

- *Leveler / Cut-To-Length* – The primary components of this line are:
 - *Leveler* – This multi-roll device applies bending forces to remove incoming stress and flatness deformations to form a “flat strip”. The leveler is a stress redistribution machine and does not achieve a complete plastic deformation of the material (only partial depth yielding). Therefore, the Leveler may leave latent (hidden) internal stress patterns, appearing as distortions in cut pieces.
 - *Shear* – This device cuts the flat sheet into specific lengths.
- *Temper / Leveler / Cut-To-Length* – This line arrangement is similar to the above noted form, but also includes an “In-Line” Temper Mill prior to the Leveler.
 - *Temper Mill* – This rolling mill applies bi-axial forces (both compression and tensile) that cause the material to experience a “full depth yielding” plastic deformation, thereby relieving the incoming stress patterns within the material, improving strip shape, providing a prescribed degree of work hardening and removing the Lüder Band producing Yield Point Region (in low carbon steels).

Feeding this tempered, stress relieved (reprogrammed) material to the Leveler provides the most ideal conditions for the Leveler to achieve a flat strip having little or no “hidden” stress.

1.1 “End User” Requirements for Cut Plates

The “End Users” of flat plate products have a variety of applications, but most involve the “patterned cuts” of parts / components for use in the manufacturing / fabrication of their products (load bearing welded structures, equipment, cosmetic shrouds and enclosures, bases and supports, etc.). The delivered plates must adhere to

geometric, flatness and internal stress requirements to facilitate the planned application of the material.

1.2 Geometric & Internal Stress Requirements

The key factor to the “End User” is the geometry and internal stress patterns of the delivered plates.

- *Geometry* – The plate should lay “still water flat” and have no flatness distortions that would compromise the “End User’s” fabrication or manufacturing process. The plate’s length must meet the specifications and have “square” corners.
- *Internal Stress Patterns* – The plate should be “stress free” and have no latent (hidden) residual internal stresses. Any remaining internal stresses (that may be present) should have no influence on the pattern cut pieces.

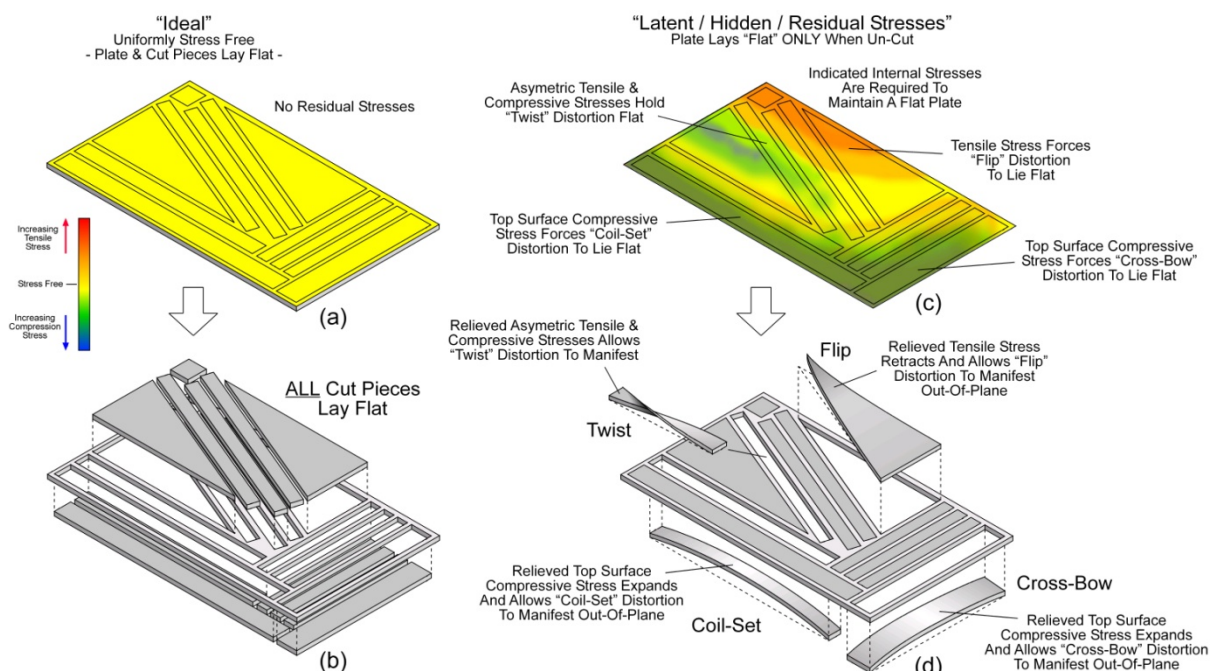


Figure 2 – Illustrations of the internal stress patterns in a perfectly flat “cut plate” product, and how internal (hidden) stresses can cause that damage to pieces “cut” from supplied flat plate.

- *Ideal Flat Plate* (Figure 2a) – The “ideal” plate contains NO residual stress, and lies perfectly flat.
- *Ideal Cut Results* (Figure 2b) – When the “ideal” plate is cut, the individual pieces remain flat.
- *Stressed Flat Plate* (Figure 2c) – A visually “flat” plate may possess latent or hidden stresses. While the plate remains “un-cut”, these coupled internal stresses interact and find an equilibrium stress field that does not buckle the plate.
- *Stressed Cut Results* (Figure 2d) – When the Stressed Flat Plate is cut, the resulting pieces may distort and not lay flat. Depending on the specific nature of the “pre-cut” internal stress distribution and cut pattern, some of the cut pieces may be subject to differential stresses that cause them to distort (out of flatness) to find their respective stress equilibriums. Essentially, the

coupled internal stresses within the “un-cut” flat plate are disrupted, causing the pieces to physically distort to find their respective equilibriums. The bottom line is that the flat plate product delivered to the “End User” must be geometrically flat and must be sufficiently stress-free, so as to not cause the cut pieces to distort.

1.3 Internal Stress & The Buckling Threshold

Only compressive forces can induce a buckling deflection. A question that comes to mind is... *How can a “flat” plate remain flat even though it has internal stresses?*

The answer has to do with the nature of the material’s Buckling Threshold.

- **Buckling Threshold** – A compressive force (or stress) threshold, F_b , that a sample piece of material can sustain (absorb / “store”) and remain in a flat equilibrium without deflecting out of plane. If an applied compressive force is greater than the Buckling Threshold, the material will be incapable of “storing” the applied stress and will deflect out of the flat plane to find an equilibrium. Figure 3 provides a sequential illustration of tensile and compressive forces applied to a plate, with the plate deflecting when the applied force exceeds the Buckling Threshold.

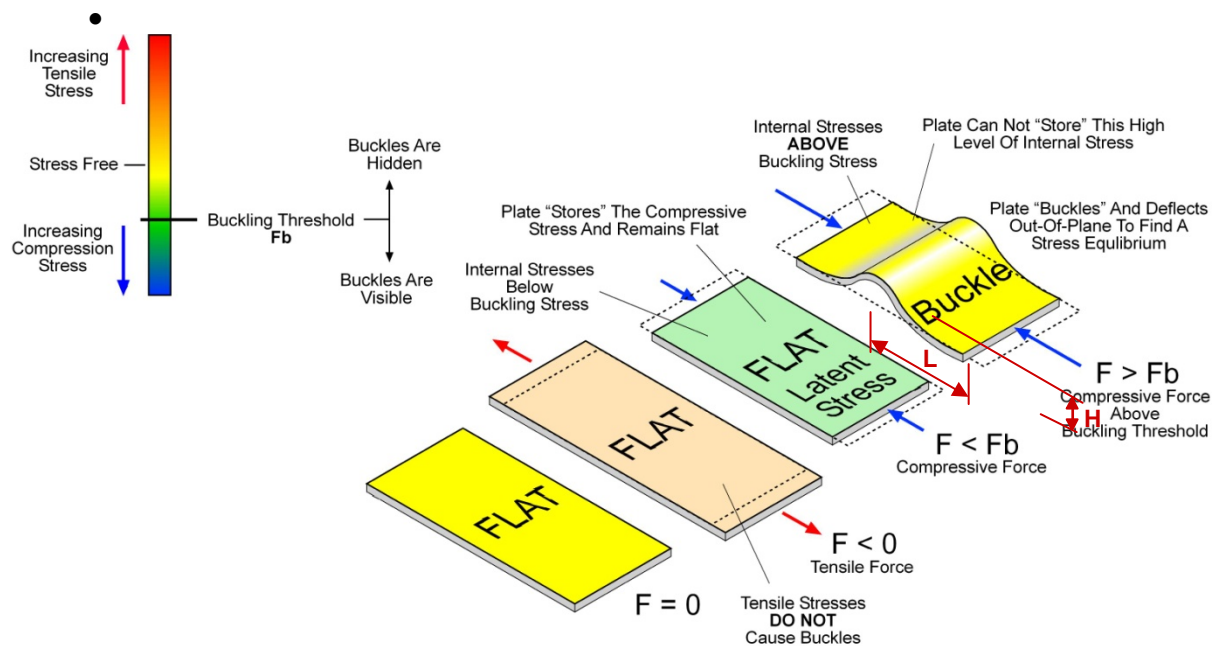


Figure 3 – Example showing how internal stresses associated with forces applied to a plate are stored or relieved by the plate deflecting out of the plane.

- **Shape Definition** – The strip shape can be expressed in several ways, including: %-Steepness and I-Units. The %-Steepness is defined as the ratio of H over L expressed in % (Figure 3). The I-unit is defined as the strip’s residual strain multiplied by 10^5 . The %-Steepness method can be measured manually using a flat reference surface, while I-Units are measured by the shape measurement device. Manual measurement of the I-Units is very tedious. By definition, the I-Unit is equal to 25 times the square of the %-Steepness.
- **Transverse Shape Disturbance** – A transverse shape disturbance is induced if the change Crown Ratio exceeds the bipolar Buckling Threshold.⁽¹⁾

$$-80\left(\frac{G}{W}\right)^{1.86} \leq \Delta\left(\frac{C_r}{G}\right) \leq 40\left(\frac{G}{W}\right)^{1.86}$$

where G is the strip thickness and W is the strip width. The crown ratio is the strip crown (center thickness – edge thickness) divided by the center thickness. The Buckling Threshold is proportional to the 1.86 power of the strip cross-section aspect ratio. “Still water flat” plates that are thinner and / or wider store less compressive stress than thicker and / or narrower plates.

2 INCOMING “HOT BAND” COIL CHARACTERISTICS

“Hot Band” coils are “rough”, full-soft / non-cold worked coils that tend to retain certain stress patterns from their formation (e.g., hot mill roll gap geometry and rolled shape, laminar cooling bed influences, coiling process, cooling process, etc.).

2.1 Stresses Associated with the Formation of Edge Waves

Edge Waves are associated with elongated strip edge regions (“over-rolled” by the hot mill) with the center of the strip being shorter. As shown in Figure 4, the plate’s internal stresses are formed by compressive forces “pushing” the long edges back to an equilibrium plane where the short centers have been stretched by tensile forces.

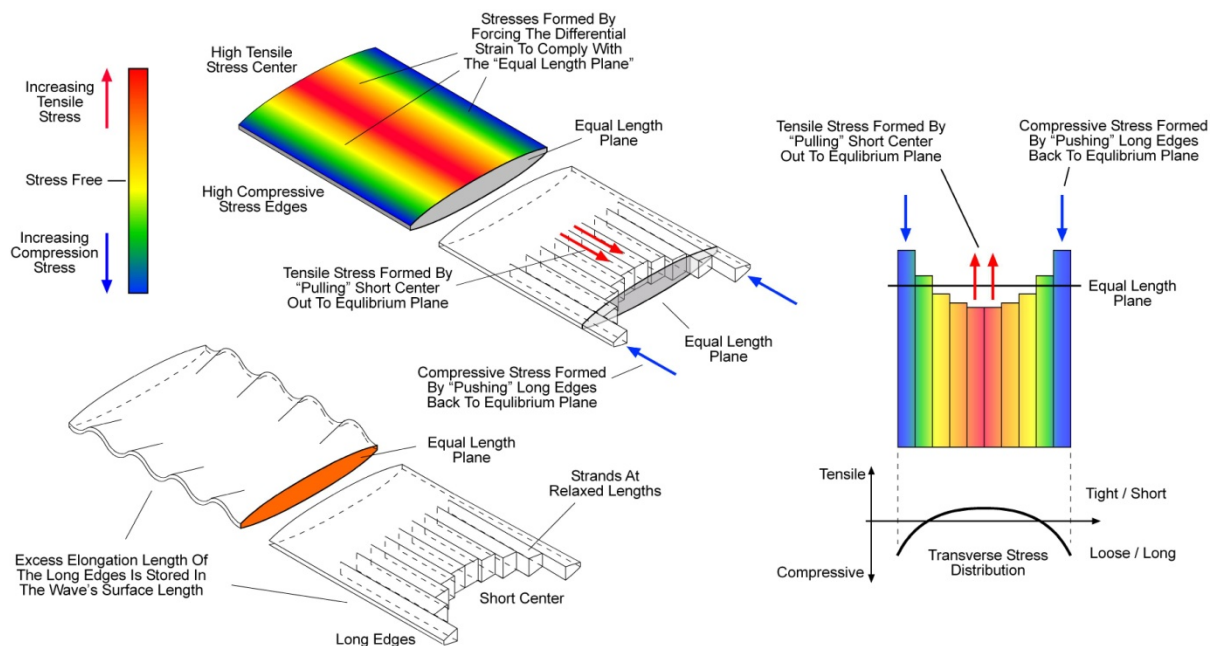


Figure 4 – Illustration of the formation of Edge Waves, including depictions of the relative strand elongations, internal stress patterns and manifest flatness distortions.

If the compressive forces on the strip edges induce stresses that exceed the local Buckling Threshold, the strip will deflect out-of-plane and essentially store the elongated edge length in the arc length of the wave.

2.2 Stresses Associated with the Formation of Center Buckles

Center-Buckles are associated with elongated strip center regions with the edges of the strip being shorter (often the result of an “over-rolled” center in the hot mill). As shown in Figure 5, the plate’s internal stresses are formed by compressive forces “pushing” the long center back to an equilibrium plane where the short edges have been stretched by tensile forces.

If the compressive forces on the strip center induce stresses that exceed the local Buckling Threshold, the strip will deflect out-of-plane and essentially store the elongated center length in the arc length of the wave.

2.3 Incoming Material Stress / Strain Curve & Temper Rolling Influence

The stress-strain curve describes the material’s reaction to an applied tensile strain, and is normally obtained from material tensile testing.⁽²⁾ Figure 6 provides a comparative example of stress / strain curves for low carbon steel and copper / brass alloys.

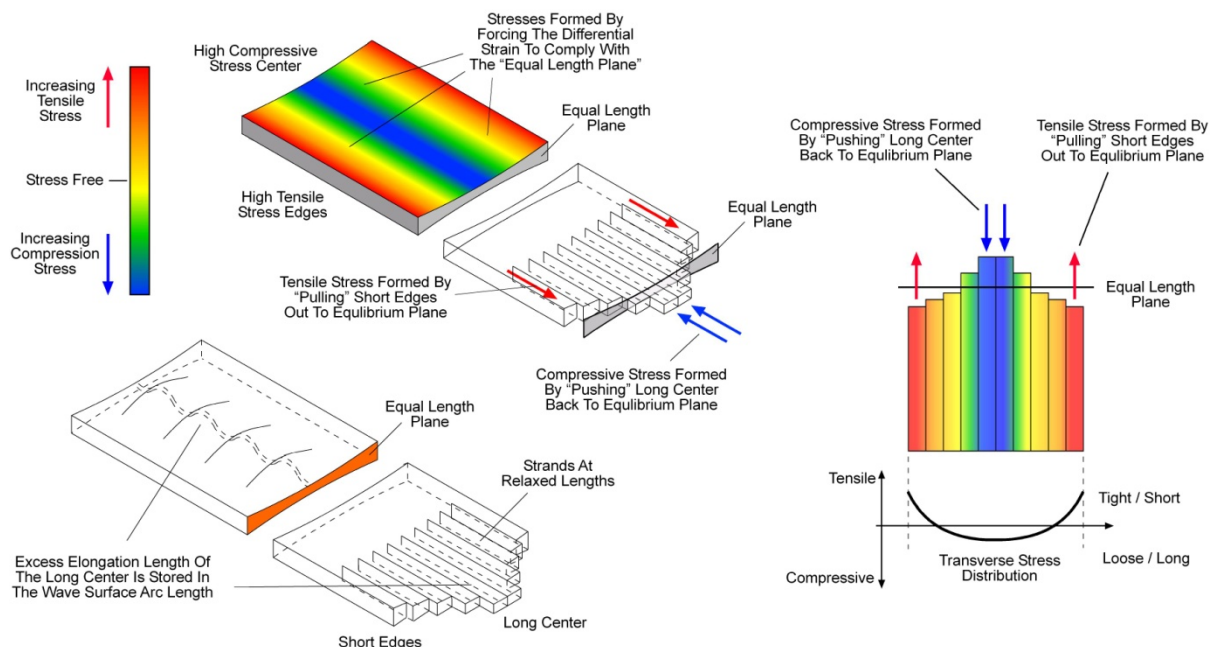


Figure 5 – Illustration of the formation of Center-Buckles, including depictions of the relative strand elongations, internal stress patterns and manifest flatness distortions.

An important distinction between these material characteristics is the presence of a Yield Point Region in low carbon steel and the smooth transition from elastic to plastic deformation in the copper alloy. The Yield Point Region is a zone of non-uniform elongation at the same applied stress. This material behavior promotes the formation of Lüder Bands (see Section 2.4).

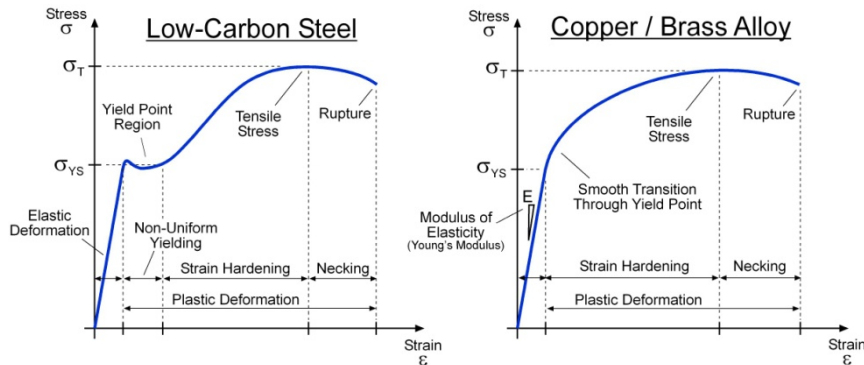


Figure 6 – Diagram showing an example of a cut pattern layout on the delivered plate. Note: These diagrams are not to scale and have been expanded at lower strains to emphasize the characteristics.

As shown in Figure 7a, with the increase in mechanical strain / elongation, the stress increases (A) linearly in the elastic zone (elastic onset) until reaching the “yield stress”, σ_{YS} , where the material undergoes plastic deformation / strain hardening (B). If the applied strain is released, the material releases the stress through an elastic recovery (C), leaving a permanent strain (permanent set).

If mechanical elongation is re-applied, the stress reaction follows a new relationship (green curve of Figure 7b). This curve has an elevated yield stress and higher tensile strength (work hardening). More importantly, this new, green curve does not have a yield point region (i.e., no ability to form Lüder Bands).

The actions and material behavior shown in Figures 7a and 7b, are the direct consequence of Temper Rolling (see Section 3.2).

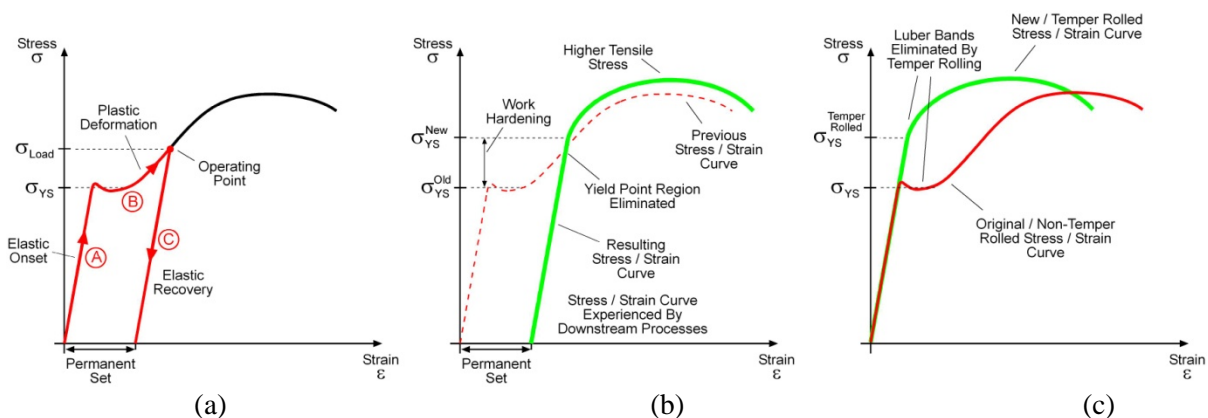


Figure 7 – Stress / strain curve examples for low carbon steel: a) Cycle of applied mechanical strain / yielding stress reaction through plastic deformation and permanent set (Temper Rolling), b) Stress / strain curve of the Temper Rolled material, c) Comparison of Temper Rolled and Non-Temper Rolled material characteristics.

Figure 7c provides a comparison between the original “Hot Band” / Non-Temper Rolled material (red) and the tempering rolled material (green). The important distinctions between these two(2) characteristics are:

- Temper Rolled material has a higher, work hardened yield stress and tensile strength
- Temper Rolled material does not possess a yield point region, and therefore will not form Lüder Bands

As will be discussed in Section 3, this is a very important attribute of Temper Rolling.

2.4 Lüder Bands

As noted in Section 2.3 and discussed in Hertzberg⁽³⁾ and Verlinden,⁽⁴⁾ low carbon and mild steels have a yield drop (following the Upper Yield Point) in which the yielded region of the material has a lower strength than their surrounding material. Here, plastic deformation is concentrated into discrete bands / regions of stress concentration, which form Lüders bands. Figure 8a shows this phenomena within the details of the Yield Point Region.

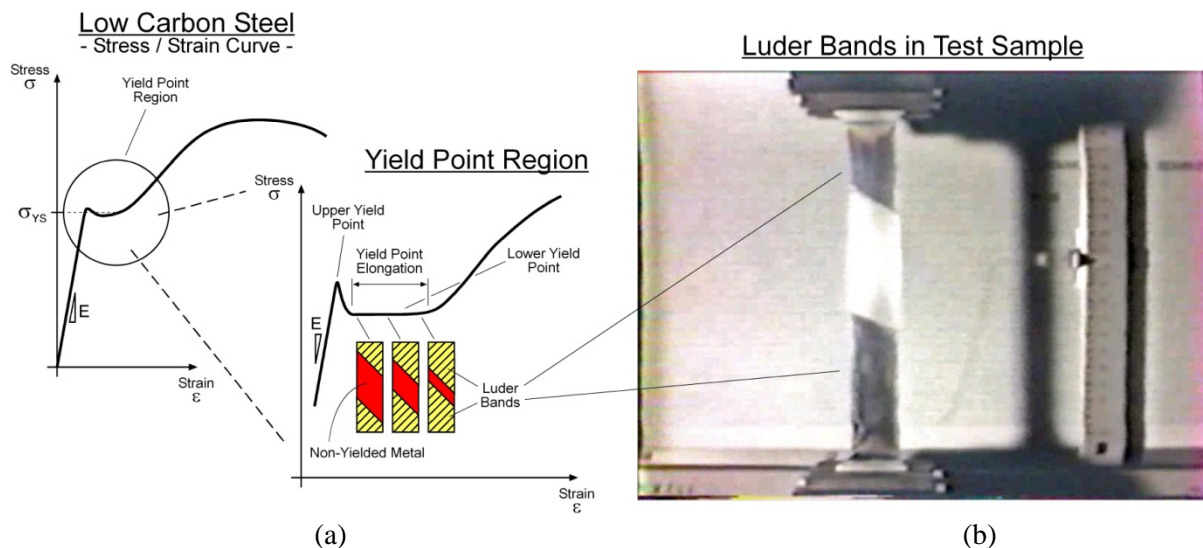


Figure 8 – Yield Point Region and the formation of Lüder Bands: a) Details of the Yield Point Region in low carbon steel and the formation of Lüder Bands in the horizontal (quasi-perfect plastic deformation zone), b) Appearance of Lüder Bands in tensile testing sample piece.

Theoretically, Lüder Band forms when the material operates in the “near-perfect plastic” zone (horizontal component) of the Yield Point Region (see Figure 8a). In this zone, the applied stress corresponds to many arbitrary strains (not in a 1-to-1 relationship). From plasticity theory, Lüder Bands are a form of stress wave propagation. Material in the Lüder Band is subjected to plastic deformation while the other area is still in the elastic state. In the Lüder Band, the elasto-plastic Navier’s equation becomes a hyperbolic-type differential equation (wave equation) in the perfect plastic zone, while other zones (elastic zone or non-perfect plastic zone) remain an elliptic-type equation (diffusion equation).⁽⁵⁾ Theoretical results can predict the initiation and development of Lüder Bands under various loading conditions and the crossing angle of the bands.

After initially yielding, Lüders Bands form from the stress application boundary, penetrate into the core portion, and eventually fail the material under load. Yet, during this process, new Lüder Bands are generated continuously between other bands until rupture. Hence, the material is easier to break if Lüder Bands are generated and propagated to the strip interior.

This is a crucial concern in addressing “End User” requirements, in that the possible formation of Lüder Bands in the delivered plate must be addressed by suppressing the Yield Point Region through Temper Rolling.

The other form of Lüder Banding is often termed “stretcher-strain” marks.^(3,4) that appear as kinks or flow lines on the surface of the material. These defect marks are unacceptable in surface finish critical material (e.g., exposed automotive body parts).

This is another area of concern to the “End User” requirements, in that the Lüder Band formation must be addressed by suppressing the Yield Point Region through Temper Rolling.

3 EQUIPMENT PROCESSING CAPABILITIES

In evaluating the two(2) possible line configurations of Figure 1 (i.e., Leveler / CTL or Temper / Leveler / CTL), it is necessary to examine the characteristics and contributions that the Leveler and Temper Mill provide to “flat” plate production.

3.1 Understanding the Leveler’s Process Capabilities

Heavy gauge, plate leveling is performed by the bending actions of Roller Leveling processes / equipment. This bending process imparts specific plastic deformation characteristics into the material. Figure 9 provides various details associated with Roller Leveling.

As shown in Figure 9a, the action of strip / plate bending generates tensile stresses in the top surface and compressive stresses in the bottom surface (in contact with the roll). It is important to note the diminishing amplitude of the bending stresses near the strip center-line. If the resulting stress (either tensile or compressive), exceeds the yield stress, that region of material will be deformed permanently, while the non-yielded core will remain in an elastic state (Figure 9b). Because the Leveler does not achieve a fully penetrating plastic deformation, the Leveler can not cause an elongation of all “strands” (Figures 4 and 5) across the strip width (as in Temper Rolling).

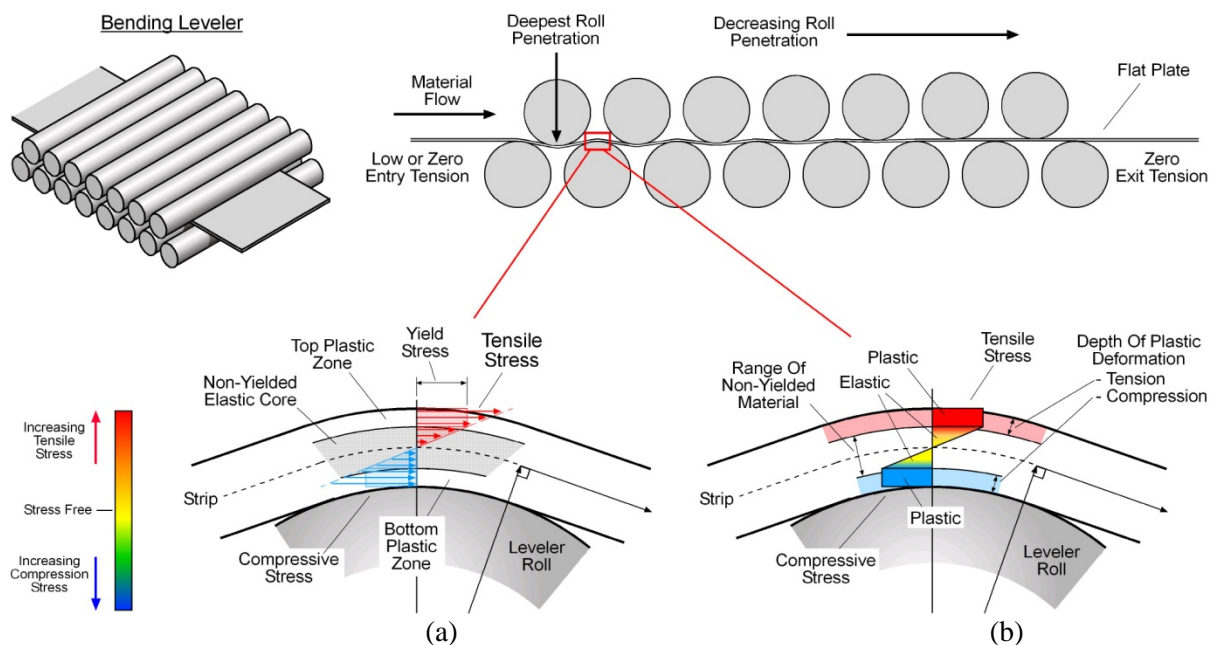


Figure 9 – Basic description of the Roller Leveling process and the manner in which the “leveling stresses” are imparted into the material through bending actions: a) Illustration of the stress distribution within the strip, b) Illustration of the elastic and plastic deformations produced by the strip / plate bending.

The Leveler produces a flat plate by elongating ONLY the shorter / tighter strands (in the incoming material), but leaving the longer / looser strands “only lightly touched”.

This is a very important factor, since this indicates that the Leveler will induce a non-uniform depth of plastic deformation across the strip width. The distribution of Leveler induced depth of plastic deformation for incoming material having edge wave distortions, is shown in Figure 10.

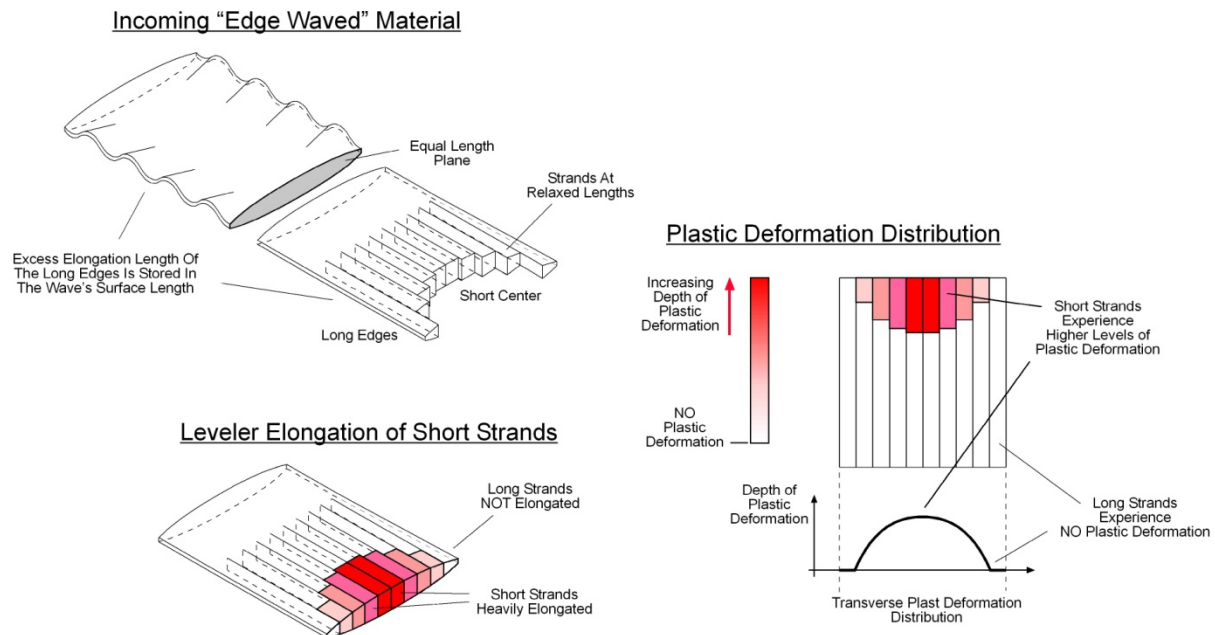


Figure 10 – Illustration of the Leveler's distribution of the depth of plastic deformation applied to edge waved incoming material to produce a flat plate. Incoming regions having short / tight strand lengths, will experience deeper plastic deformation, while the longest incoming strands will experience no plastic deformation

Another related factor is associated with the extent of yielding that the strands will experience. As discussed in Section 2.4, if the material is not strained beyond the Yield Point Region, the resulting material will be highly susceptible to Lüder Band formation. Because the Leveler applies different levels of plastic deformation, the various strand lengths will operate at different locations on the stress / strain curve. Figure 11 illustrates the different stress / strain curve operating points and the likelihood for the occurrence of Lüder Bands on incoming edge waved material.

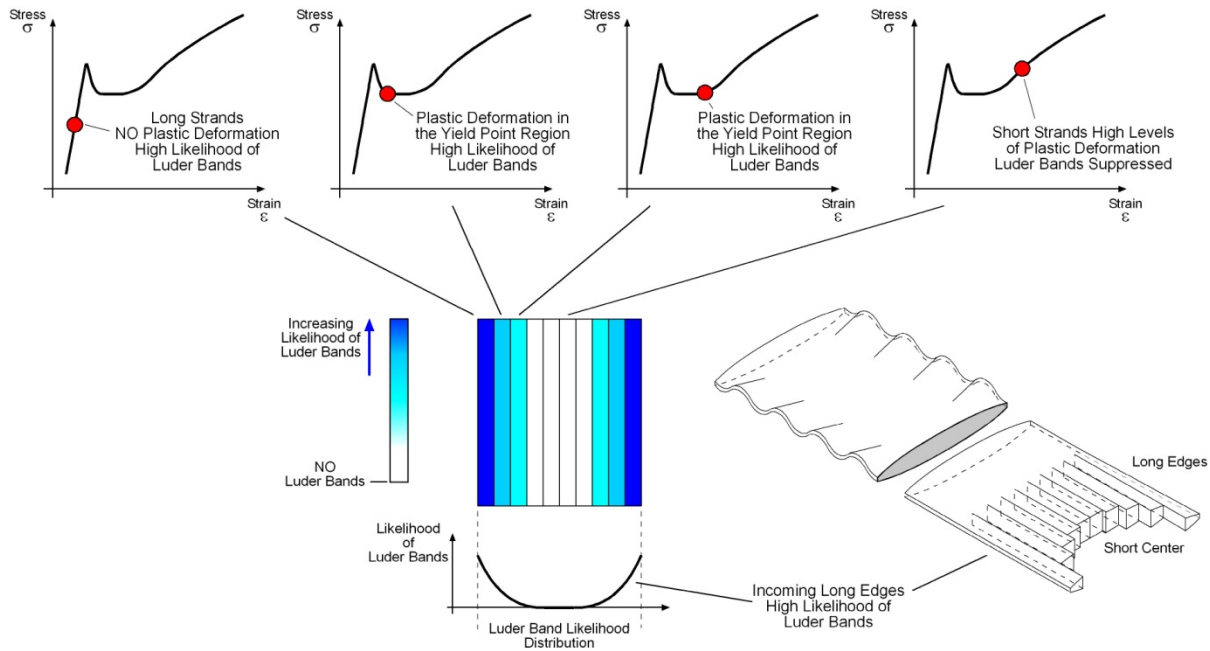


Figure 11 – Illustration of the Leveler's distribution along the material width having the potential to generate future Lüder Bands, for incoming edge waves.

The important issue to note in Figure 11 is the way that longer incoming strands (those that will not be worked / elongated by the Leveler) are not yielded beyond the Yield Point Region of the stress / strain curve. These longer incoming strands will still retain certain unsatisfactory aspects of the Yield Point Region, and will therefore have a high likelihood of producing Lüder Bands in the “End User's” fabrication / manufacturing processes.

The same can be said for incoming center buckled material, where the leveled strip / plate center / quarter region may mistakenly appear to have a uniform temper. In processes like pressing, drawing or bending, Lüder Bands (of the stretcher strain form) will appear damaging the structural integrity or surface quality of the processed / manufactured parts.

3.1.1 Summary of analysis

The Stand-Alone Roller Leveler has inherent limitation and provides the “End User” with material that will possess non-uniform and inconsistent characteristics.

- The Leveler does not achieve a fully penetrating plastic deformation through the material thickness.
- Long incoming strands will experience little or no plastic deformation.
- Long incoming strands will experience yielding that may not extend beyond the Yield Point Region, thereby having a high likelihood of producing future Lüder Bands in the End User's products.

3.2 Understanding the Temper Mill's Process Capabilities

In the context of this application, the primary roles / purposes of the Temper Mill are:

- Provide Yield Point Region suppression by yielding the material well into the plastic deformation zone to remove the possibility of generating Lüder Bands, and achieve better formability
- Provide cold work hardening (make lower yield strength materials stronger).

- Provide full strip thickness plastic deformation penetration to completely reorganize (reprogram) the strip stress distribution
- Use the Temper Mill's shape correction actuators to apply repairing corrections to the incoming "Hot Band" coil strip shape
- Prepare the strip shape to better feed and operate the downstream Leveler

The characteristics of the Temper Rolled material exiting the mill (green curve in Figure 12a) differs significantly (in a positive sense) from the "Hot Band" material entering the mill (red curve in Figure 12a). As shown in Figure 12b and 12c, the Temper Mill achieves this by simultaneously applying bi-axial stresses (both compressive and tensile) in the roll bite. Entry material is drawn into the roll bite and experiences elastic deformation as the geometry of the strip / work roll interface constrict (A). The progressive constriction of the roll bite geometry applies additional stress which ultimately yields the material, and the material "flows" in plastic deformation (B) toward the exit side of the roll bite. As the material emerges from the roll bite, it experiences a rapid drop in applied stress and elastically recovers (C) to its permanent set.

During Temper Rolling well beyond the Yield Point Region, the plastic deformation penetrates the strip completely, causing a new stress pattern and strand elongation to be generated (completely transformed from the differing entry material). The resulting Temper Rolled material is work hardened and has no Yield Point Region (can generate NO Lüder Bands).

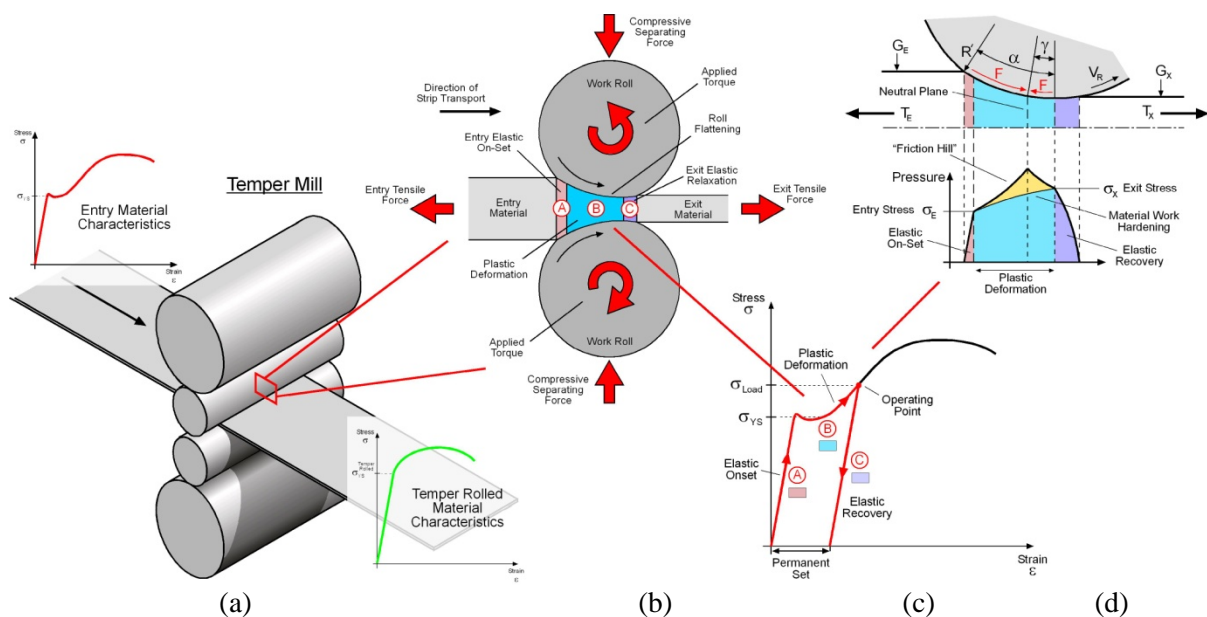


Figure 12 – Basic description of the Temper Rolling process: a) Illustration of the changes in material characteristics by Temper Rolling, b) Longitudinal cross-section of the roll bite showing the bi-axial stress application and reactions by the material and work rolls, c) Stress / strain curve progression of Temper Rolling, d) Roll bite compressive separating force pressure distribution.

3.3 Process Capabilities of the Temper Mill / Leveler Combination

The In-Line Temper Mill (upstream of the Leveler) expands the overall Leveler / Cut-To-Length line's capabilities to achieve the "End User's" requirements.

- The Temper Mill's full strip width elongation action (typically 1% to 2.5%) creates slightly longer finished coil, providing more material to cut
- The Temper Mill fully yields the material and removes the Yield Point Region

- The Leveler receives material that will not form Lüder Bands
- The Temper Mill forms a plastic deformation zone that fully penetrates the strip / plate thickness
 - The stresses of the incoming “Hot Band” coil distortions are removed
 - The Leveler’s limited depth of plastic deformation penetration is assisted by the Temper Mill “preparing” the material for the Leveler
- The Temper Mill work hardens the material
 - Higher yield strength material is formed and the light temper will not lose the ductility of the material
 - Value is added to the “End User’s” product by transforming lower grade (lower yield strength) steels to higher yield strength materials
- The Temper Mill’s shape correction actuators are used to “prepare” the strip shape to optimize Leveler operations
 - The Leveler receives “improved” material (over the “Hot Band” coil)
 - Tight edges and a slightly over-rolled center promote good threading, alignment and tracking within the Leveler
 - The amplitude of the Leveler’s incoming shape is within the Leveler’s shape adjustment capability

Figure 14 provides an illustration of the Temper Mill / Leveler arrangement and shows the advantages and expansion of the process capability through the inclusion of the upstream In-Line Temper Mill.

4 CONCLUSION: COMPARISON OF PROCESSING CAPABILITIES

This paper has addressed the question as to how the contributions of an In-Line Temper Mill improve the process performance and capabilities of heavy gauge, Leveler / Cut-To-Length lines. We can conclude this discussion by making a comparative analysis of the two line arrangements.

4.1 Limitations of the “Stand-Alone” Leveler

The “Stand-Alone” Leveler has distinct limitations in its ability to achieve the “End User’s” requirements:

- The Leveler’s bending actions don’t achieve a full penetration plastic deformation
 - The resulting material is not yielded through its thickness
 - An elastic core remains in the flat, leveled plates
 - The elastic core retains the stress patterns of the incoming material
 - These retained stress patterns will contain residual, latent / hidden incoming “Hot Band” coil distortions
 - These distortions may “reappear” in the “End User’s” application (pieces cut from a seemingly “still water flat” plate)
- The Leveler does not work harden the material
- The Leveler does not induce a uniform depth of plastic deformation
 - Short incoming strands of the material are elongated to match the lengths of the longer strands
 - Long incoming strands may not experience sufficient plastic deformation
- The Leveler does not remove the Yield Point Region in low carbon steels
 - Lüder Bands may form in the “End User’s” material

- The likelihood of Lüder Bands occurring will be on the longer incoming strands (those regions where incoming waves / buckles are located – edge waves, center buckles, etc.)

Figure 13 illustrates these limitations by showing how incoming “Hot Band” coil distortions are not completely removed from the “still water flat”, cut plate stress patterns.

The retained stresses are hidden / latent and have a lower amplitude than the overall plate’s Buckling Threshold. During later work (by the “End User’s” manufacturing / fabrication process), localized dormant stress patterns may appear as flatness distortions (twist, flip, warping, bow, etc.) in the pieces cut from the plate by the “End User”. Lüder Bands may appear in subsequent bending or forming processes, creating unacceptable surface defects and potentially weakening the “End User’s” manufactured part.

These limitations in the “Stand-Alone” Leveler / Cut-To-Length line’s ability to produce flat plates that meet the “End User’s” requirements, must be carefully considered in the decision to apply this form of process line arrangement.

4.2 Advantages of the In-Line Temper / Leveler / Cut-To-Length Configuration

Incorporating a Temper Mill in a Leveler / Cut-To-Length line has distinct advantages and provides an expansion of the overall line’s process capabilities. These advantages and capabilities advancements provide the ability to achieve the “End User’s” requirements:

- The Temper Mill’s full strip width elongation action (typically 1% to 2.5%) creates a longer finished coil, providing more material to cut.
- The Temper Mill yields the material and removes the Yield Point Region
 - A new, Temper Rolled material stress / strain curve is produced
 - The Leveler receives material that will not form Lüder Bands
- The Temper Mill forms a plastic deformation zone that fully penetrates the strip / plate thickness
 - The stresses associated with the incoming “Hot Band” coil distortions are removed
 - The Leveler’s limited depth of plastic deformation penetration is assisted by the Temper Mill
- The Temper Mill work hardens the material
 - High yield strength material is formed
 - Value is added to the “End User’s” product by transforming lower grade (lower yield strength) steels to higher yield strength materials
- The Temper Mill’s shape correction actuation “prepares” the strip shape to optimize Leveler operations (the Leveler receives “improved” material when compared to the “Hot Band” coil)
 - Tight edges and a slightly over-rolled center promote good threading, alignment and tracking within the Leveler
 - The amplitude of the Leveler’s incoming shape is within the Leveler’s shape adjustment capability

Figure 14 illustrates these advantages and expansions of the process capability by showing how incoming “Hot Band” coil distortions are completely removed by the fully penetrating plastic deformation within the Temper Mill’s roll gap.

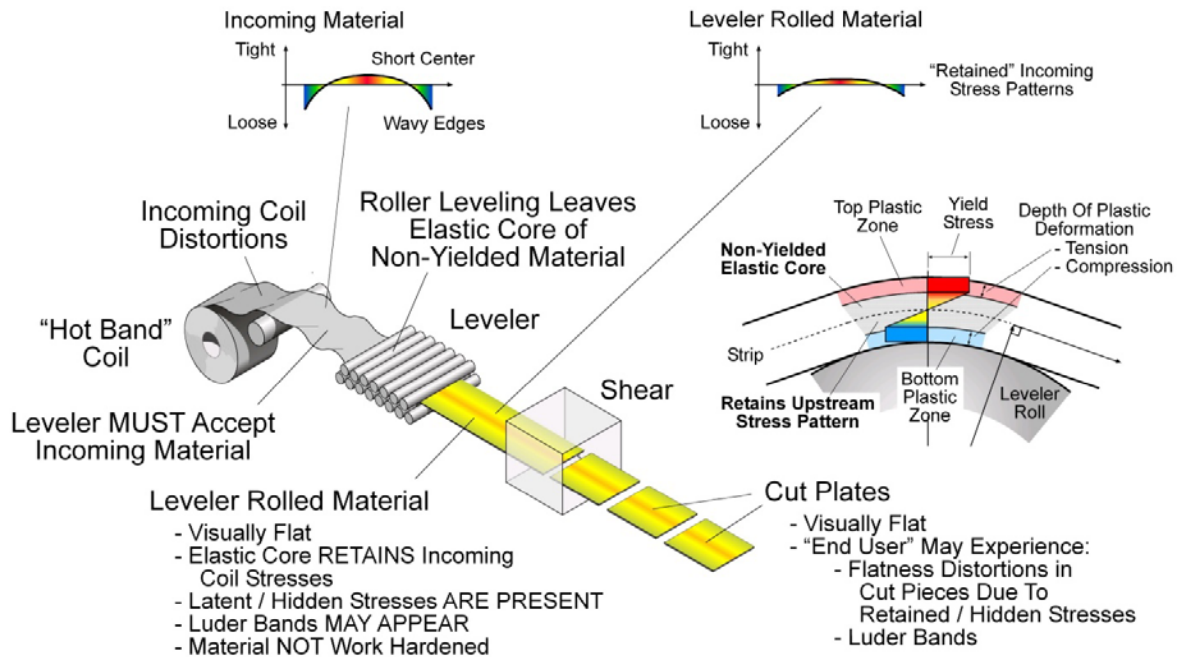


Figure 13 – Illustration of the "Stand Alone" Leveler's process capabilities and limitation.

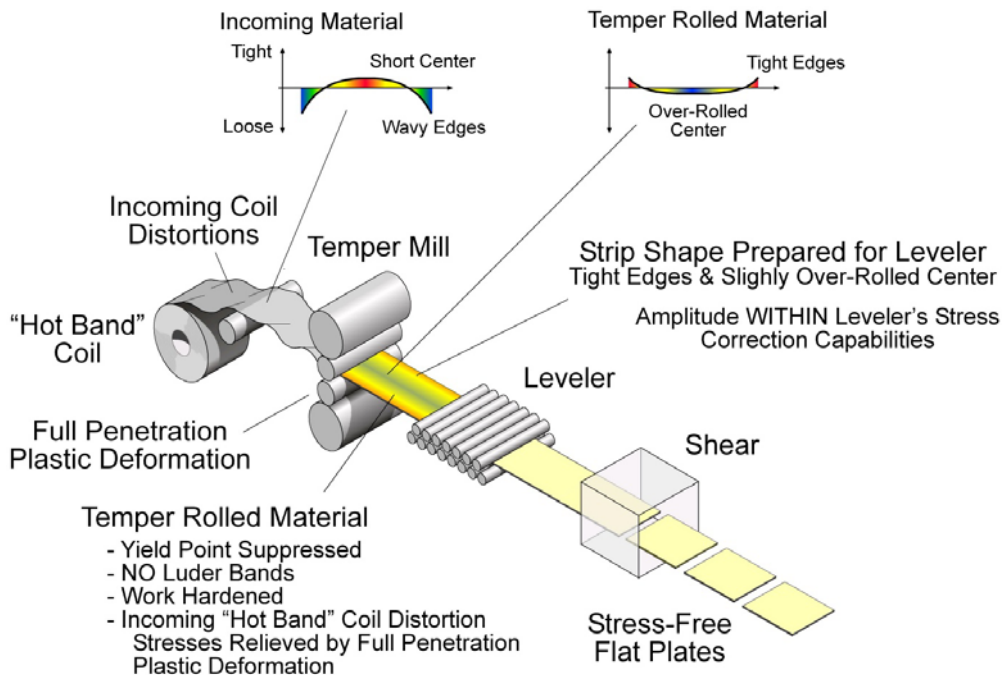


Figure 14 – Illustration of the Temper Mill / Leveler advantages and expanded process capabilities

4.3 Conclusions Reached

Based on this analysis, the conclusion drawn is that the inclusion of an In-Line Temper Mill provides important benefits and greatly assists the Leveler in achieving the "End User's" requirements. The combined Temper / Leveler / Cut-To-Length is far more robust in its ability to contend with incoming "Hot Band" coil distortions (when compared to a "Stand-Alone" Leveler arrangement). The Temper Rolled, finished cut plates will be of greater quality and value to the "End User".

As "End User" specifications and quality requirements become tighter and more stringent, the In-Line Temper Mill becomes a necessary component in the line.



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