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CHARACTERIZATION OF THE RESTRICTIONS IN SHAPE ADJUSTMENT CAPABILITIES ASSOCIATED WITH ACTUATOR CONSTRAINTS[1]

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Abstract

The shape actuators of 20-high cluster mills (top crown eccentrics and laterally traversing, tapered 1st intermediate rolls) have specific physical and operational constraints (step, bending, penetration depth limits, etc.) that diminish their ability to produce/ deliver the desired shape adjustments. These limitations can be viewed as restrictions or bounds on the set of desired shape targets that the mill can achieve. The behavior of the shape actuators can be characterized with influence functions (spatial waveforms). Using orthogonal polynomials as a basis set, these functions are described by a distribution/ spectra of spatial curvatures. The actuation constraints form bounding envelopes on the distribution/ spectra, which indicates the shape adjustments that the mill can and cannot provide. This paper examines and defines the curvature distribution bounding envelopes for the cluster mill's top crown eccentric shape actuators for the classical "step limit" constraints. It is shown that these shape corrections capabilities envelopes are also functions of the material geometry, yield stress and reduction scheduling of the mill. Further, the nature of the roll cluster set-up and backing assembly BC actuation shafts (solid, flexible, segmented idler roll) are examined to determine their impact on these envelopes.

Key words: 20-high cluster mills; Shape actuators; Actuator constraints; Bounding envelopes.

Resumo

Os atuadores do controlador de forma do laminador *20-high* (excêntricos do coroamento superior, lateral transverso e cônico dos rolos primeiros intermediários) têm restrições físicas e operacionais específicas (passo, dobramento, limites de profundidade de penetração, etc.) que diminuem sua capacidade de produzir / entregar os ajustes da forma desejada. Estas limitações podem ser vistas como restrições ou limites sobre o conjunto da forma desejada como meta que o laminador pode atingir. O comportamento dos atuadores de forma pode ser caracterizado com funções de influência (onda de forma espacial). Usando polinômios ortogonais como o conjunto de base, essas funções são descritas por uma distribuição de curvaturas espaciais. As restrições da distribuição de atuação dos envelopes de forma, indica os ajustes de forma que o laminador pode e que não pode produzir. Este artigo analisa e define a distribuição da curvatura delimitadora dos envelopes para os atuadores de coroamento excêntricos do laminador cluster e as restrições para o "passo limite" clássico. Mostra que a capacidade de correção destes envelopes de forma também são funções da geometria do material, do limite de elasticidade e do plano de redução planejado para o laminador. Além disso, a natureza do *set-up* do cluster e a montagem dos eixos de atuação dos rolamentos de encosto BC (sólido, flexível, *idler roll* segmentado) são examinados para determinar seu impacto sobre esses envelopes.

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1 INTRODUCTION

20-High Cluster Mills adjust the strip's shape by coordinating a set of actuators to provide corrective changes in the applied transverse pressure distribution (across the width of the roll gap) that modify the localized strip elongations, thereby altering the strain patterns, and subsequently the transverse stress/ tension distribution of the rolled strip. Each actuator induces a unique stress adjustment pattern on the strip's transverse stress distribution that can be characterized as a continuous spatial influence function. A typical shape actuator arrangement(1) involves top crown eccentrics (B&C Backing Assemblies) and the laterally traversing, tapered 1st Intermediate Rolls (1st IMRs) (Figure 1). This figure includes the typical waveform characteristics of the individual actuator's spatial influence functions.

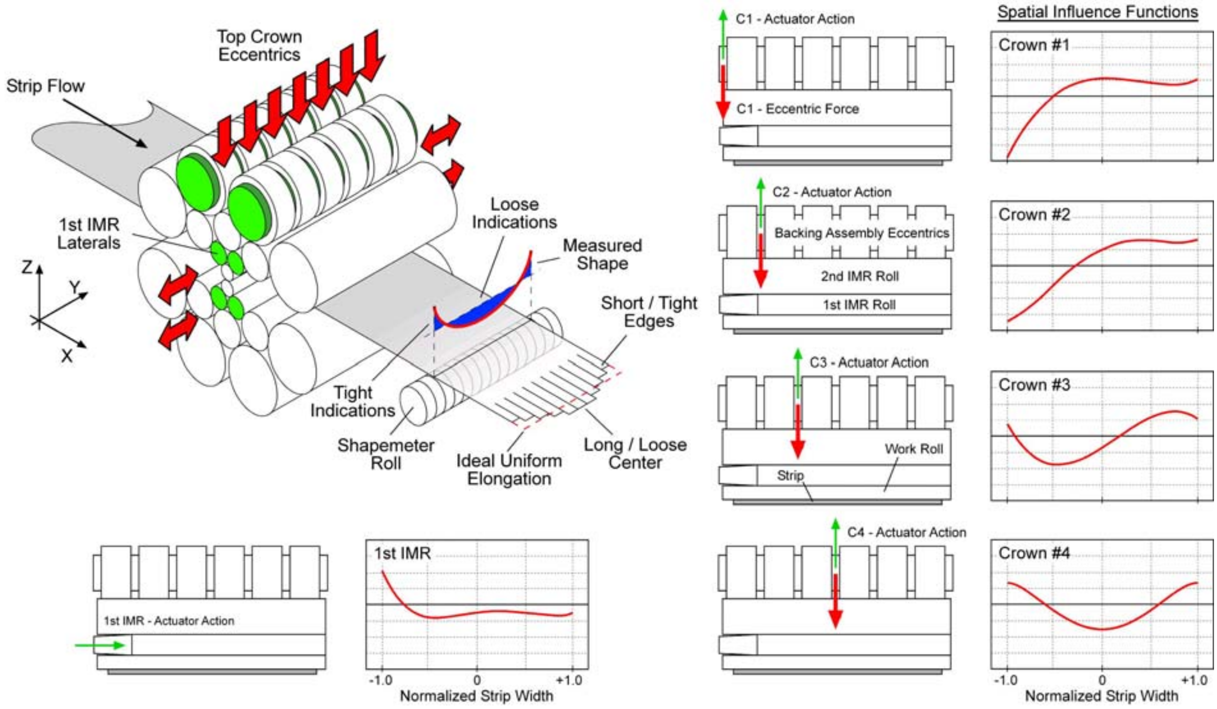

Figure 1. Illustration of a 20-High Cluster Mill and shape actuators, along with diagrams showing the typical characteristics of the actuator induced transverse stress patterns (Spatial Influence Functions).

1.1 Discrete Spatial Model

It is possible to describe the transverse stress pattern of the rolled strip, by the combined effects of the actuators influences, the incoming strip stress pattern and the deformation characteristics of the roll cluster under separating force load.(2,3) Based on a discrete spatial model, this vector relationship is shown in Figure 2 and given by:

$$\mathbf{S}(y_M) = \mathbf{S}_0(y_M) + \mathbf{S}_R(y_M) + \mathbf{S}_A(y_M) \qquad (1)$$

$\mathbf{S}(y_M)$ Rolled, exist strip shape / stress pattern (2a)

$\mathbf{S}_0(y_M)$ Incoming strip shape / stress pattern (2b)

$\mathbf{S}_R\left(y_M\right)$ Natural, mechanical deformation characteristics of the roll cluster while under separating force loading (2c)

$\mathbf{S}_A\left(y_M\right)$ Shape / stress pattern induced by the actuation system (2d)

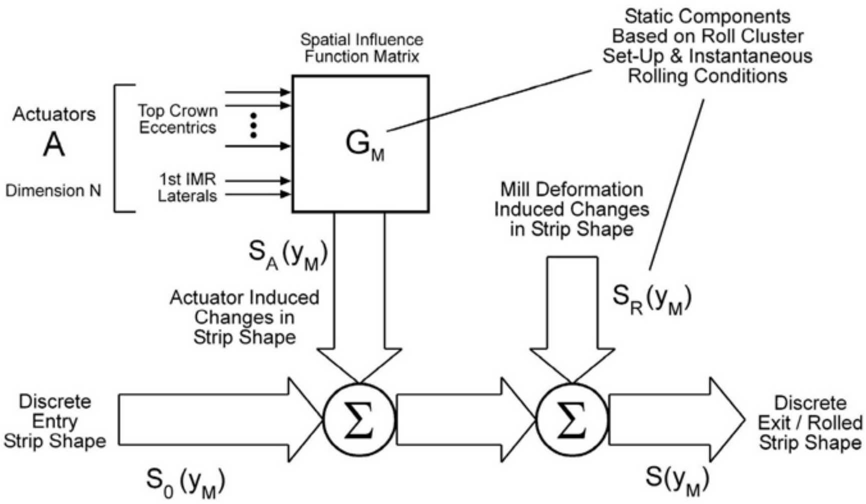

Figure 2. Block diagram illustration of the discrete spatial model.

The discrete spatial variable, y_M, is an M dimensional set of uniformly distributed locations across the strip width.

$$y_M = \left\{ y_M^0, y_M^1, \cdots, y_M^{M-1} \right\} \quad y_M^0 = -1 \quad y_M^{M-1} = +1 \tag{3}$$

This represents a normalized Sobolev space mapping across the strip width, W, having a domain interval [-W/2, +W/2] $\rightarrow$ [-1, 1].

1.2 Shape Actuation Model

The dynamic shape actuation contribution, $S_A(y_M)$, is represented by the matrix multiplication of the N dimensional shape actuation vector, $A \in \Re^N$, onto the spatial influence function matrix, $G_M \in \Re^{MxN}$, which is given by some autors:(2-6)

$$\mathbf{S}_A\left(y_M\right) = \mathbf{G}_M \mathbf{A} \tag{4}$$

Were:

$$\mathbf{A} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N-2} \\ a_{N-1} \end{bmatrix} = \begin{bmatrix} L_T \\ \text{—} \\ \bar{\mathbf{C}} \\ \text{—} \\ L_B \end{bmatrix} = \begin{bmatrix} L_T \\ C_1 \\ C_2 \\ \vdots \\ C_{N-2} \\ L_B \end{bmatrix} \begin{matrix} \text{Top Lateral} \\ \text{Crown \#1} \\ \text{Crown \#2} \\ \vdots \\ \text{Crown \#(N-2)} \\ \text{Bottom Lateral} \end{matrix} \tag{5}$$

The columns of the matrix, G_M, are the evaluations of the spatial influence polynomials at the sampling grid associated with y_M.(7,8)

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$$\mathbf{G}_M = \left[\mathbf{G}_M^0(y_M) \,\middle|\, \mathbf{G}_M^1(y_M) \,\middle|\, \mathbf{G}_M^2(y_M) \,\middle|\, \cdots \,\middle|\, \mathbf{G}_M^{N-2}(y_M) \,\middle|\, \mathbf{G}_M^{N-1}(y_M) \right] \tag{6}$$

The actuator spatial influence functions can be derived from analytic modeling[9-11] or through direct on-line evaluation of the mill's response characteristics.[8,12]

1.3 Roll Cluster Deformation

Under separating force loading, the mill deformation characteristic, $S_R(y_M)$, is a function of the roll diameter profiles within the roll cluster, the flexibility of the roll cluster[12] and the instantaneous rolling conditions (including separating force, rolling speed, strip width, thickness and yield stress). In general, $S_R(y_M)$, is a static vector composed of primarily of 2nd and maybe 4th order spatial curvatures, that can not be modified while rolling operations are underway.

1.4 Available Degrees of Freedom in Adjusting the Rolled Strip Shape

Together, the actuated dynamic component, $S_A(y_M)$, and the static component, $S_R(y_M)$, constitute the available degrees of freedom in adjusting the rolled strip shape, both actively and as part of a process engineering design (i.e., the selection of the roll cluster set-up and tuning of the pass schedule separating forces).
As will be discussed in the next section, the shape actuation induced stress patterns, $S_A(y_M)$, are fundamentally constrained by both physical limitations and operational / protective constraints. These constraints limit the extent of the actuators ability to dynamically address deviations between the rolled / exit strip shape and the desired shape target, $S_T(y_M)$, during rolling operations.
The design of $S_R(y_M)$ is a complex "black art" that involves paying careful attention to the cluster's roll profiles and pass schedule, to program the progression of the mill deformation over the pass-to-pass sequence. The important aspect of this shape adjustment component, is that it can only be modified by exchanging rolls within the cluster, and not during rolling operations.

1.5 Discussion

The mill's shape actuation capabilities are fundamentally limited, and the mill can only provide a restricted range of modifications to the strip shape/ transverse stress pattern. Essentially, the constrained actuation system can only form and apply a bounded family of transverse waveform patterns (a bounded envelope).
Using the above discrete spatial model, it is possible to evaluate the characteristics of this bounded family of shape actuated waveform patterns (bounded shape adjustment envelope) and thereby determine the manner in which the shape actuation constraints impact the mill's ability to adjust the strip shape.
The transverse waveform patterns of the strip shape and actuator influences are highly coupled, non-linear functions, that provide little or no insight into the underlying characteristics/ conditions that form the bounds of the shape adjustment envelope.

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It is possible to describe the spatial waveform patterns in terms of their basic orders of linearly independent curvatures (1st, 2nd, 3rd functions, etc.).[2,3] Using this approach, the seeming complexities of the various waveform patterns can be simplified to vectoral distribution (or spectrum) of curvatures (a unique vector for each and every spatial waveform).
By forming a Cartesian coordinate system (in terms of the fundamental curvature components), it is possible to express the bounded family of shape actuated waveforms as a closed / bounded region in the space of this coordinate system.
The remainder of this paper examines the nature of the constrained shape actuation capabilities in terms of these bounded regions (the shape adjustment envelopes).

2 SHAPE ACTUATION SYSTEM CONSTRAINTS

The constraints on the top crown eccentrics and 1st IMR laterals are fundamentally different, and need to be examined separately.

2.1 1st IMR Constraints

As shown in Figure 3, there are five primary constraints on the 1st IMR actuators.[1-3]

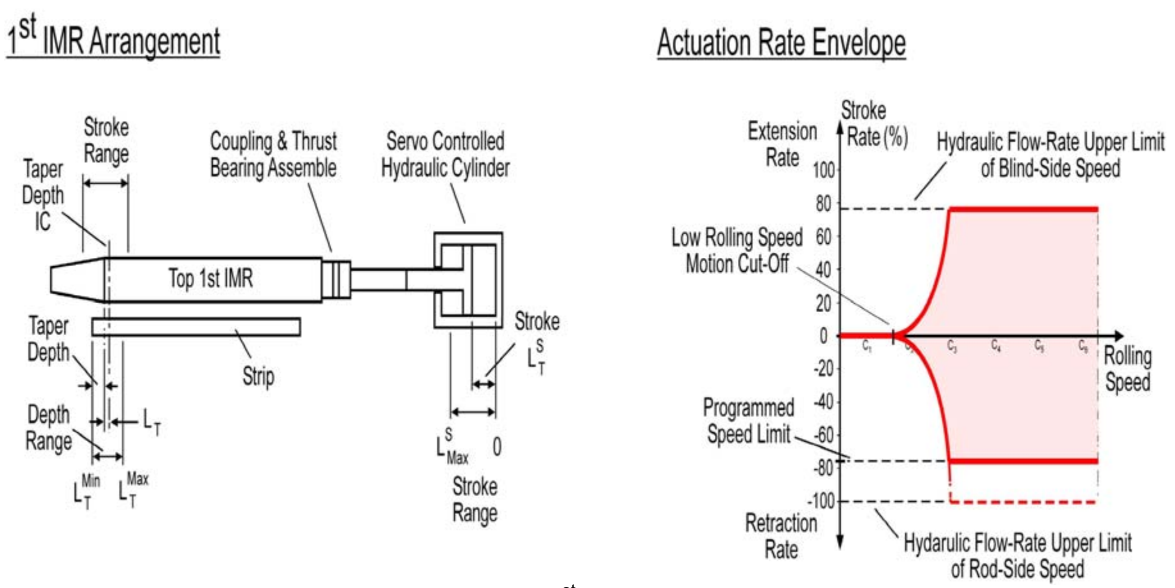

Figure 3. Illustration of the 1st IMR actuation and constraints.

2.1.1 Hydraulic cylinder stroke limits

The driving hydraulic cylinders have physical stroke limits.

2.1.2 Taper depth limits

The taper depth describes how far the taper knee penetrates the strip edge. The initial conditions on the taper depth are often pass schedule preset and these floating, hard operations limits may be asymmetrical.

2.1.3 Rolling speed restriction

At low rolling speeds, the inter-roll, surface contact frictions, experienced by the 1st IMRs (3 per roll), are too great to allow lateral movement without inducing excessive stresses on the rolls' mechanisms (i.e., couplings, thrust bearings, etc.). It is therefore

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necessary to impose a position holding action while rolling below a specified speed. This is a purely operational constraint that causes the 1st IMRs to transition from a dynamic to a static actuator.

2.1.4 Separating force restriction

As part of the above noted rolling speed restriction, the instantaneous separating force is included, since it is a major contributor to the friction loading experienced by the roll surface.

2.1.5 Stroke rate limit

Like the rolling speed restriction, this lateral motion speed limit is intended to not induce excess force/ stress damage of the roll's mechanisms. This limit is typically an increasing function with rolling speed, and ultimately reaches a specified maximum allowable speed or hydraulic fluid flow rate limit (often to induce stroke rate equality in the presence of the cylinder area differentials – to provide identical bipolar speeds).

2.2 Top Crown Eccentric Constraints

As shown in Figure 4, there are four primary constraints on the top crown eccentric actuators.[1-3]

2.2.1 Hydraulic cylinder stroke & working range limits

The top crown eccentrics operate in a working range within the physical stroke limits of the hydraulic cylinders, associated with the proper meshing of the actuator's rod-end rack and the gear teeth of the crown eccentric rings within the backing assembly saddles. The working range forms the stroke positioning limits during normal operation.

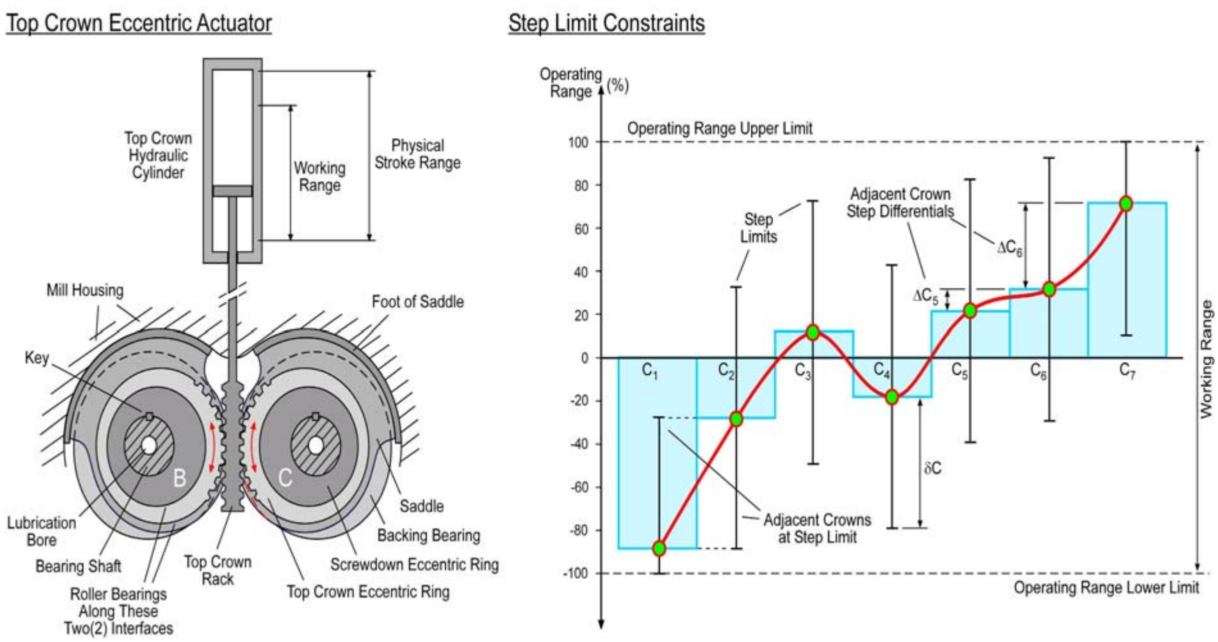

Figure 4. Illustration of the top crown eccentric actuation and constraints.

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2.2.2 Adjacent crown step limits

These are relative constraints (1st order bending moments) that are imposed to protect the backing assembly shafts and bearings from excess, localized shear stresses, which could lead to more rapid bearing wear and / or failure. The constraint restricts the differential positions of adjacent actuators via:

$$\left|\delta C_i\right| = \left|C_{i+1} - C_i\right| \leq \left|\delta C_{Max}\right| \quad \text{for } i=1,2,\ldots,N\text{-}3 \tag{7}$$

This constraint can vary significantly, depending on the flexibility of the backing assembly shafts.[1,12] Smaller mills (ZR24 class) can have relatively tight constraints (δC_{Max} = 25% of the working range), while larger mills (ZR22 class) with flexible backing assemblies and segmented idler rolls, can have no step limit restrictions.

2.2.3 Bending limits

These constraints place limits on the extent of the 2nd order bending moment (i.e., limits the amount of "zig-zag" that can be introduced). Similar to the step limits, these constraints are imposed to protect the backing assembly shafts and bearings from excess stresses. The constraint restricts the differential positions of adjacent actuators via:

$$\left|\delta^2 C_i\right| = \left|C_{i+2} - 2C_{i+1} + C_i\right| \leq \left|\delta^2 C_{Max}\right| \quad \text{for } i=1,2,\ldots,N\text{-}4 \tag{8}$$

2.2.4 Stroke rate limit

The top crown eccentrics have no rolling speed restrictions or required stroke rate limits, however rate limits are often imposed to compensate for the hydraulic fluid flow limits (often to induce a stroke rate equality in the presence of the cylinder area differentials – to provide identical bipolar speeds).

3 SPATIAL CHARACTERIZATION THROUGH PARAMETER DECOMPOSITION

The spatial waveform patterns of the strip shape (incoming & rolled), the shape actuation and roll cluster deflection can be described by a simplifying vectoral distribution/ spectrum of spatial curvatures.[2,3] An interesting Approximation Theory[13] approach uses an orthogonal polynomial basis to frame the description of the shape patterns/ waveforms. Here, the spatial characteristics of the shape patterns/ waveforms are described by the combination of polynomial-based spatial curvatures.

In the appropriate Sobolev space, the set of Gram orthogonal polynomials[2,3,5,14,15] form a complete basis set. Therefore, functions occurring in the continuous domain -1 ≤ y ≤ 1 can be expressed by the truncated expansion:

$$\mathbf{S}(y) = \sum_{i=1}^{Np} \$_S^i \mathbf{P}_i(y) \tag{9}$$

This parameterization of the waveform patterns is based on the combination of Np degrees of monotonically increasing orders of spatial curvature. The vector collection, $\$_S$, of the coefficients provides a means of consolidating the parameterization of the describing spatial curvatures (forming a distribution/ spectrum of curvature contributions).

$$\mathbf{\$_S} = \left[\$_S^1 \quad \$_S^2 \quad \cdots \quad \$_S^{Np}\right]^T \tag{10}$$

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Over the discrete spatial sampling of y_M, and the inner product nature of Eq(9), a matrix transformation relationship can be formed[2,3] from orthogonal polynomials evaluations:

$$\mathbf{S} = \tilde{\mathbf{P}}\,\$_S \qquad \Leftrightarrow \qquad \$_S = \left(\tilde{\mathbf{P}}^T\tilde{\mathbf{P}}\right)^{-1}\tilde{\mathbf{P}}^T\mathbf{S} = \tilde{\mathbf{P}}^T\mathbf{S} \tag{11}$$

Where the matrix $\tilde{\mathbf{P}}$ is the Curvature Transform Matrix, and because of the polynomial orthogonality, the Inverse Transform Matrix is the transpose. As shown by Zipf and Godwin,[2,3] the general relationship of Eq(1), in terms of spatial curvatures, becomes:

$$\mathbf{\$}_S = \mathbf{\$}_0 + \mathbf{\$}_R + \tilde{\mathbf{P}}^T\mathbf{G}_M\mathbf{A} \tag{12}$$

Figure 5 provides a block diagram illustration of this parameter composition/ decomposition transformation.

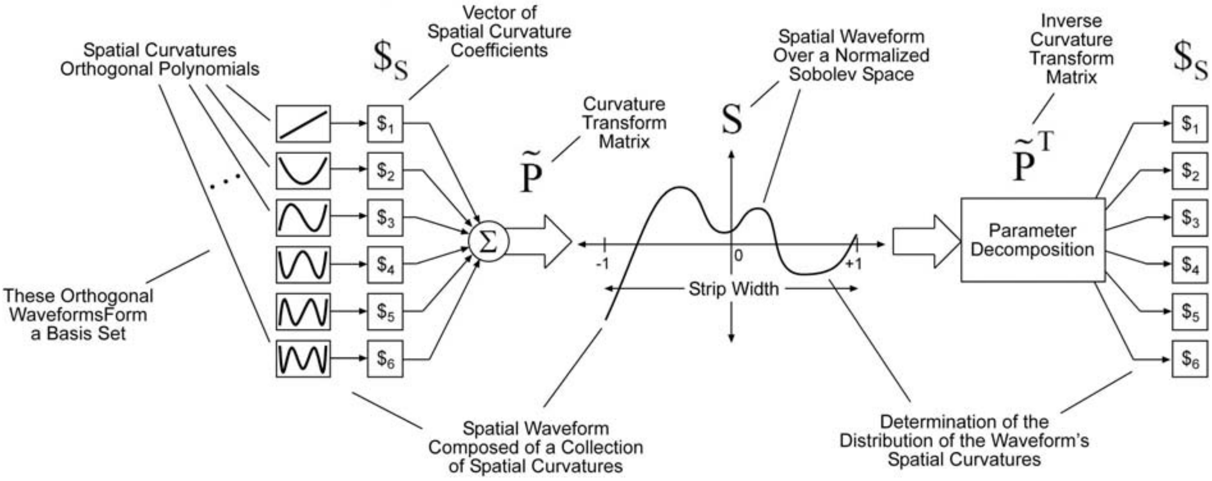

Figure 5. Block diagram showing the parameter composition/ decomposition process.

It is possible to expand on this process by forming a linearly independent, Cartesian coordinate system (associated with the nature of the orthogonal polynomials that form the decomposition), which uses the basis directions of the spatial curvature vector, $\$_S$. Figure 6 provides an illustration of this type of coordinate system.

In Figure 6, the spatial waveform, S, is decomposed into it constituent curvature components, $\$_S$. This distribution/ spectrum of spatial curvatures uniquely describes the spatial waveform, in terms of the amount of each orthogonal polynomial needed to replicate the waveform. The resulting representation is a vector (point) within this Cartesian space, leading to a simplified means of characterizing the attributes of the spatial waveform.

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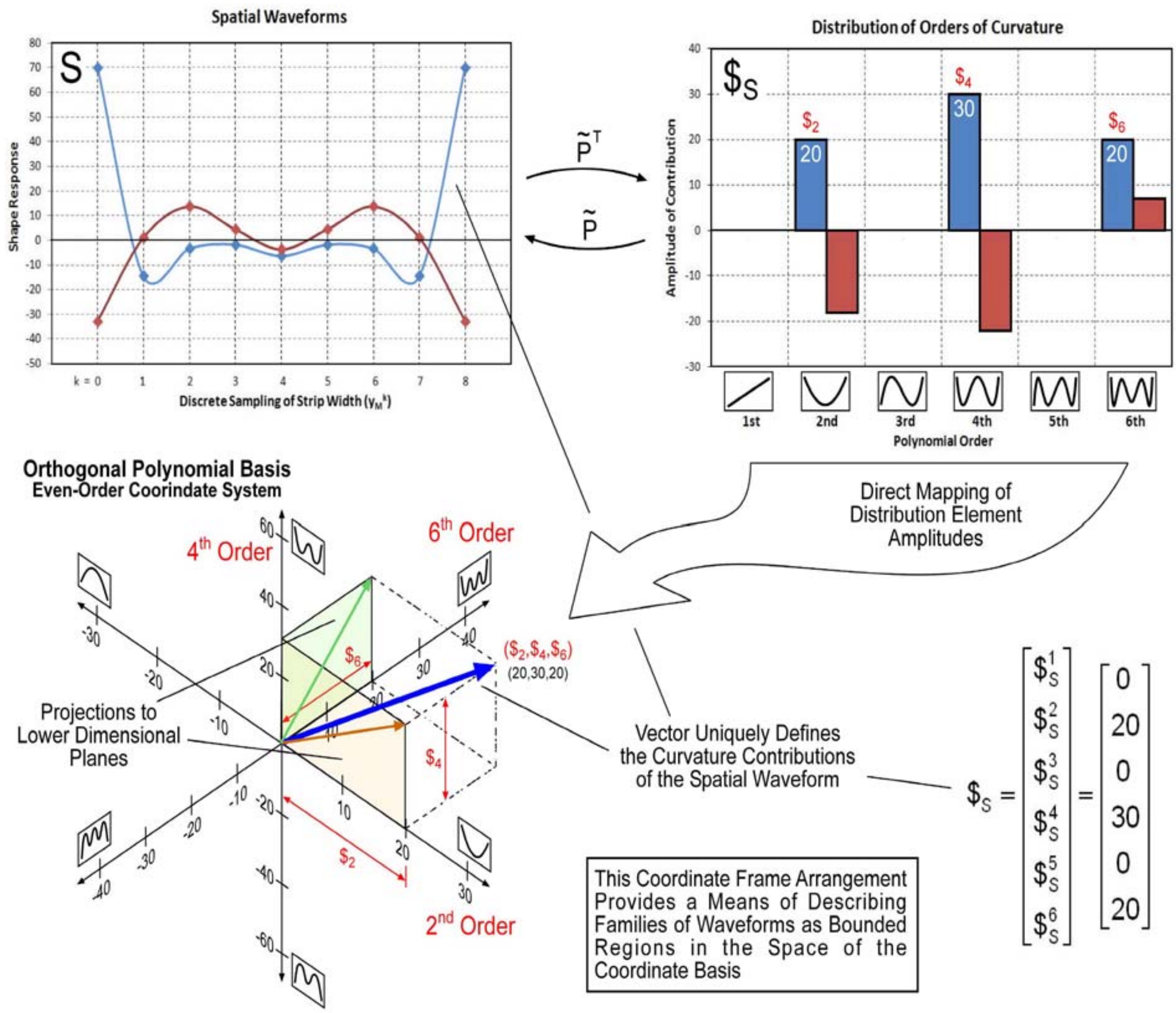

Figure 6. Illustration of the linearly independent Cartesian coordinate system formation and the associated mapping of a given spatial waveform's distribution / spectrum of spatial curvatures to form a single vector (point) within this linear space.

4 FORMING SPATIAL CURVATURE ENVELOPES

A Spatial Curvature Envelope is defined as the set of vectors (points) within a spatial coordinate system (typically of even/ symmetric orders – Figure 6), that are associated with a family of spatial waveforms.
The family of spatial waveforms is associated with ALL the possible combinations of actuator settings that comply with the following conditions/ constraints:

- stroke limit – to comply with the physical cylinder stroke and/or working range;
- zero mean – to comply with the requirement that the crown actuation be non-interactive with the automatic gauge control (AGC) system;
- step limit – this provides a compliance with the 1st order bending moment protection of the backing assembly bearings and shafts.

The process of determining the Bounding Spatial Curvature Envelope involves identifying ALL combinations of the constrained actuation, and the resulting spatial waveforms associated with the adjustments to the strip shape, S_A. These waveform patterns are decomposed to their curvature distributions/ spectra, $\$_A$. For this study, only the even/ symmetric waveforms and spectra were considered, and therefore only those spectra having zero (or sufficiently small) odd terms were used. The resulting curvature spectra components are mapped to the curvature coordinate system (a point plotted for each acceptable waveform). As the entire set of admissible actuation patterns (family of admissible waveforms) are evaluated, a set of points develops and "fills-in" a region in the curvature coordinate system plane. After all possible actuation patterns are evaluated, image processing methods (involving vertex and extremity locating techniques) are used to determine the bounding envelope of the entire family of plotted points. Figure 7 provides a process flow chart and associated diagrams shown how the bounding envelope is determined.

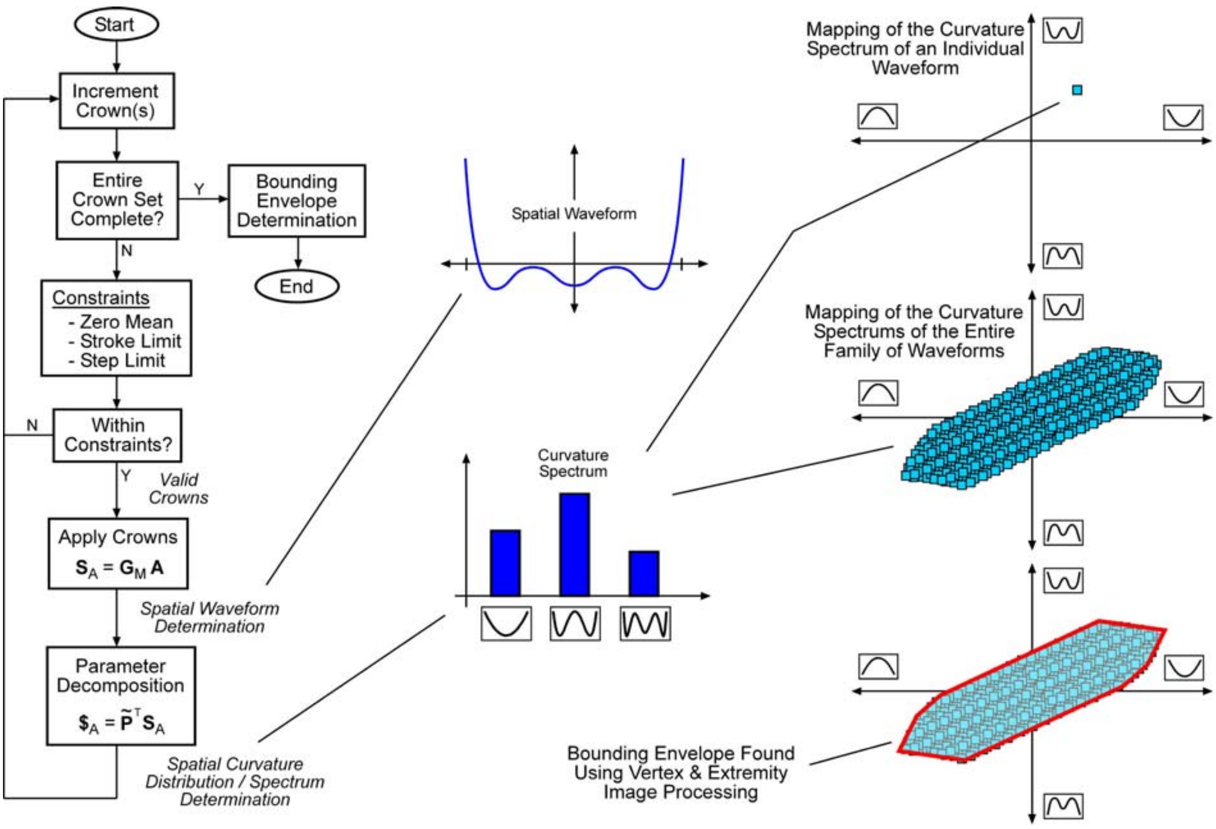

Figure 7. Process of determining the bounding shape adjustment envelope of a family of waveforms associated with the constrained top crown eccentric actuation.

In this study, the pattern of applied top crown eccentric settings had a resolution of 1% (over a range of +/-100%), which resulted in over 1.33 x 10^{16} (i.e., 201^7) possible combinations for a classical 7 crown arrangement. This extent of evaluation required (on an average), nearly 11 hours of computation on a Quad Core processor for each bounding envelope to be determined.

5 ANALYSIS & COMPARISON OF SHAPE ADJUSTMENT ENVELOPES

The process described in Section 4.0 was applied to a variety of situations and conditions of a modelled ZR23-26 class mill rolling stainless steel materials.(12) The model was formed from an adjustable composite of 3 mills that employed backing assemblies having solid and flexible shafts (FSBA), and one that included a segmented 2nd intermediate idler roll (SIR). The roll cluster was configured and arranged to optimally roll 530 mm wide strip.

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5.1 Variations in Top Crown Eccentric Actuator Step Limits

Figure 8 provides a plot of the bounding envelopes that describe the shape actuation contributions to the rolled strip stress pattern, $\$_A$, for varying levels of Step Limits, when using FSBAs and a solid top idler roll, on the cluster's optimal strip width.

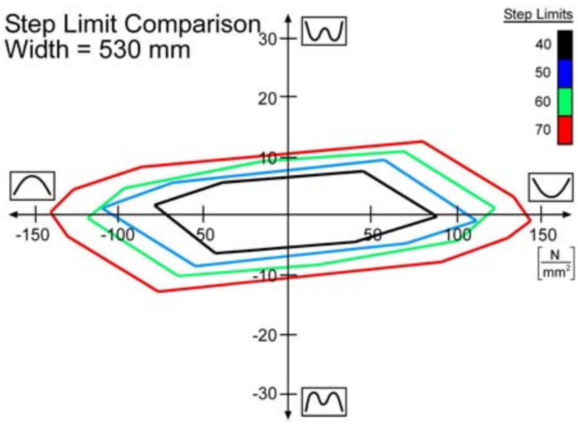
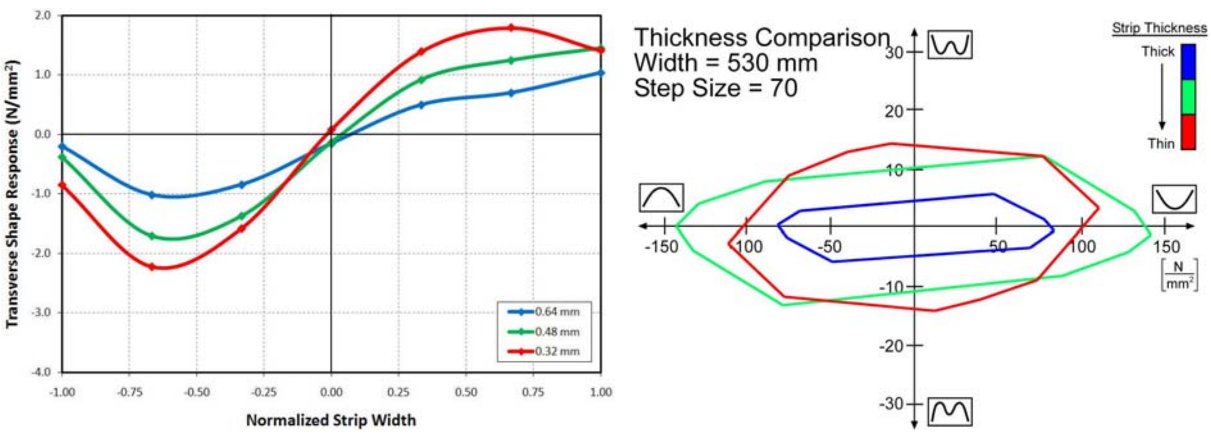


Figure 8. Bounding envelopes of the spatial curvatures associated with varying top crown eccentric actuator step limits on an optimal material width for the given roll cluster set-up.

As shown in Figure 8, the increase in the Step Limits cause decernable expansion of the bounding envelope, meaning that a greater degree of shape adjustment capability is realized with larger Step Limits. It is interesting to note that this expansion is dominated by the 2^{nd} order curvatures. This is expected, because the roll cluster's transmission of the actuation patterns (to the roll bite) behaves like a spatial low pass filter,[2,3] which suppresses high order spatial frequencies (allows the 2^{nd} order terms to prevail over the 4^{th} order components).

5.2 Variations in Strip Thickness

Figure 9 provides a plot of the bounding envelopes that describe the shape actuation contributions to the rolled strip stress pattern, S_A, for varying levels of strip thickness with the Step Limits set at 70%, on roll cluster's optimal strip width.



(a) (b)

Figure 9. Spatial waveforms and bounding curvature envelopes associated with variations in strip thickness at the optimal strip width for the roll cluster set-up with the Step Limits set at 70%: a) Crown #3 influence functions, b) Bounding envelopes.

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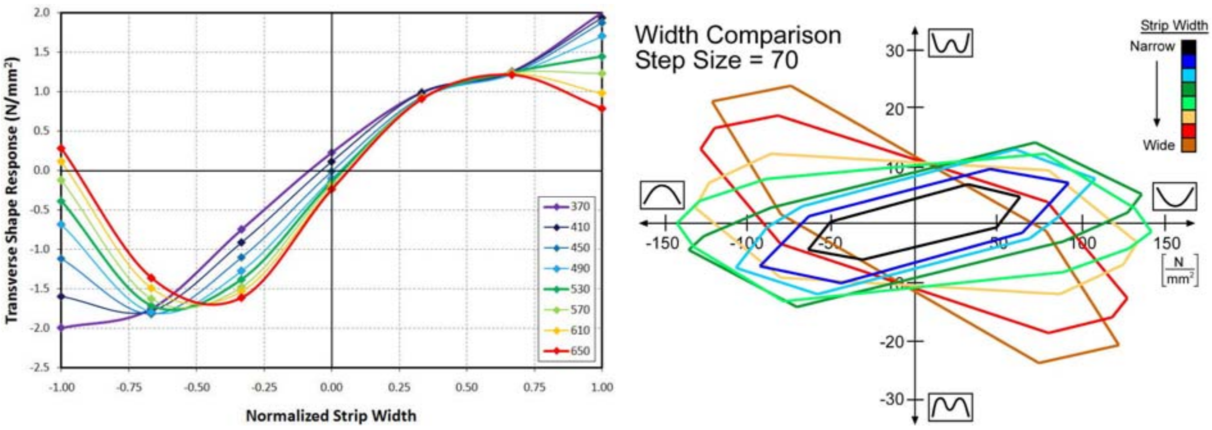
As shown in Figure 9, thicker strip is more resistant to shape actuation corrections. As the strip thickness is reduced, the envelope expands in a uniform manner. During thin strip rolling, the 2nd order terms are suppressed, while there is a clear expansion in the 4th order characteristics. This is associated with an increased cluster deflection in regions away from the location of the actuator, as the strip aspect ratio is reduced.

5.3 Variations in Strip Width

Figure 10 provides a plot of the bounding envelopes that describe the shape actuation contributions to the rolled strip stress pattern, S_A, for varying levels of strip width with the Step Limits set at 70%.



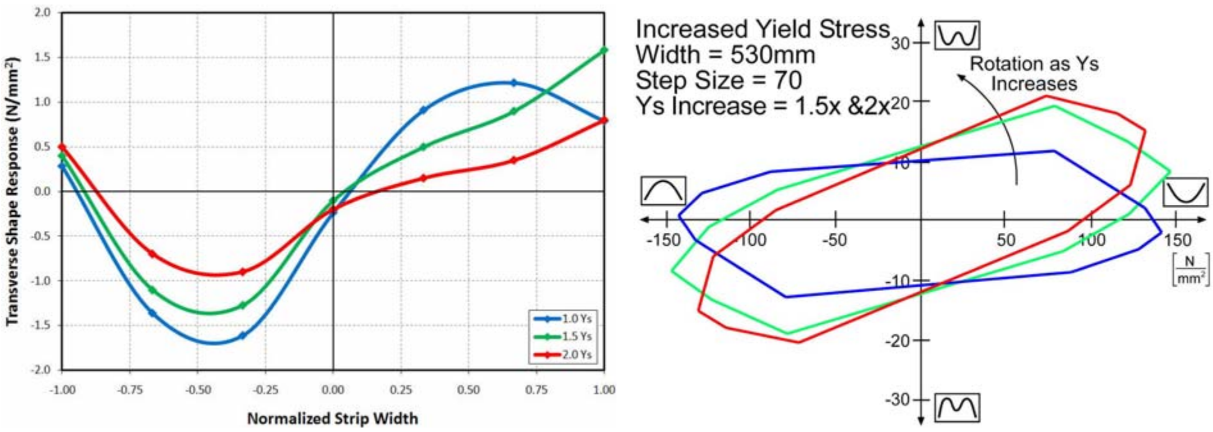
(a) (b)

Figure 10. Spatial waveforms and bounding curvature envelopes associated with variations in strip width with the Step Limits set at 70%: a) Crown #3 influence functions, b) Bounding envelopes.

As shown in Figure 10, the bounding envelope expands and rotates as the strip width increases. The expansion is associated with a greater actuator resolution on wider strip, and the ability to induce higher order adjustment curvatures (Figure 10a). The rotation characteristic is a purely geometric phenomena associated with the physical alignment of the crown actuators with the strip (Figure 10a).

5.4 Variations in Material Yield Stress

Figure 11 provides a plot of the bounding envelopes that describe the shape actuation contributions to the rolled strip stress pattern, S_A, for varying levels of strip yield stress with the Step Limits set at 70%.



(a) (b)

Figure 11. Spatial waveforms and bounding curvature envelopes associated with variations in strip yield stress with the Step Limits set at 70%: a) Crown #3 influence functions, b) Bounding envelopes.

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As shown in Figure 11, the bounding envelope does not expand, but only rotates as the strip yield stress increases. This is associated with a degree of the strip's increased resistance to shape adjustment, and also due to an increased level of 4th order involvement associated with a suppression of roll cluster deformations away from the actuator location (note the linear behaviour on the right side of Figure 11a).

5.5 Variations in Backing Assembly Shaft Type

Figure 12 provides a series of plots showing the bounding envelopes that describe the shape actuation contributions to the rolled strip stress pattern, S_A, for different types of backing assembly shafts (solid, FSBA) and 2nd intermediate top idler rolls.

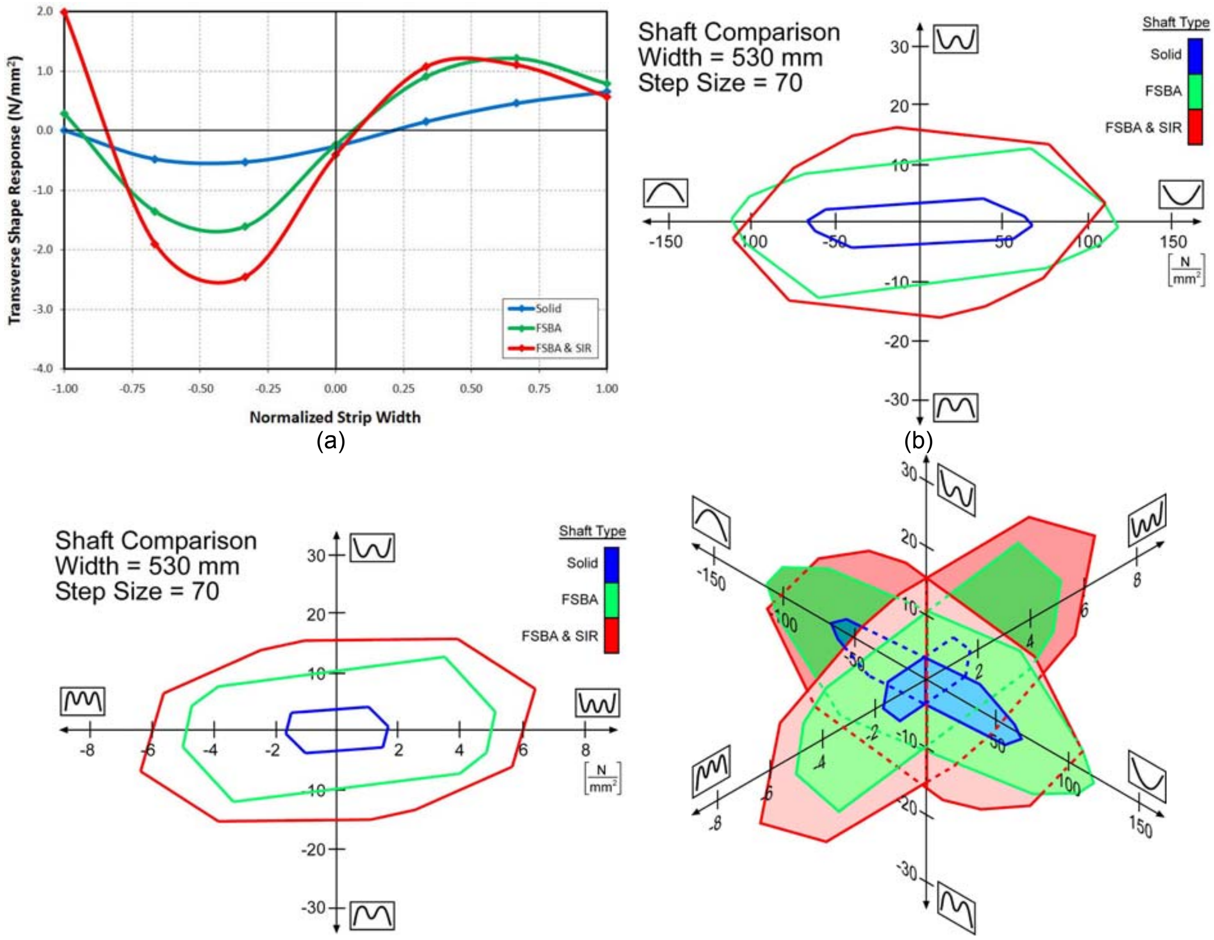

Figure 12. Spatial waveforms and curvature envelopes associated with variations in the backing assembly shafts at the optimal strip width for the roll cluster set-up: a) Crown #3 influence functions; b) 2nd and 4th order bounding envelopes; c) 4th and 6th order bounding envelopes; d) 3-Dimensional composite view of the 2nd, 4th and 6th order bounding envelopes.

As shown in Figure 12a, the solid shaft has a very rigid, non-responsive behaviour, which forms a very restricted degree of range of adjustment capabilities (see the bounding envelops of Figures 12b and 12c). The incorporation of FSBAs radically changes the spatial dynamics (Figure 12a) and expands the bounding envelopes (Figures 12b and 12c). Most notably, this enhanced flexibility expands the 6th order characteristics by nearly a factor of 3x (Figure 12c), which provides for a vast improvement in the ability to apply complex shape adjustment patterns.

The incorporation of an SIR further expands the bounding envelopes (mainly in the 4^{th} and 6^{th} order components), but the extent of this enhancement is not as drastic.
Figure 12d illustrates these flexibility enhancements in terms of a volumetric depiction. The important aspect of this diagram is the massive extent of the actuation capabilities that are gained by the use of FSBA shafts. It is true that the use of an SIR provides additional shape actuation benefits, however, the higher operational commitments of the SIR (primarily in terms of special roll grinding methods) and concerns of the segments marking the 1^{st} IMRs, may outweigh these relatively minor enhancements.

6 CONCLUSION

This paper has examined the nature and impact of shape actuator constraints on the shape adjustment capabilities of 20-high cluster mills. Using an orthogonal polynomial based parameter decomposition technique, the distribution/ spectrum of the shape adjustment spatial waveforms of the actuation influences has been obtained. It has been shown that this spectrum can be mapped to a spatial curvature describing coordinate system, where families of spatial waveforms can be depicted as a set of points within a region. Bounding envelopes are formed (about these sets of points) to define the extents of the constrained shape adjustment capabilities in terms of the range of achievable spatial curvatures. This is the primary theoretical result of this work.
Using this method, the extent of the shape adjustment capabilities of a particular class of 20-high cluster mill (ZR23-26) has been examined. Here, the variations in the shape adjustment capabilities in the presence step limit constraints, variations in strip width, thickness and yield stress have been presented and discussed. The interesting effects of type of backing assembly shaft (solid and flexible (FSBA)) and the presence (or not) of a segmented top idler roll (SIR) has been shown. The results indicate that there is a major enhancement in the dexterity of the shape actuator capabilities by employing an FSBA (as opposed to a conventional solid shaft). The use of an SIR further improves the capabilities, but the major benefit is realized with the FSBA. This is the primary practical results of this work.

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