SIMULATING A PILOT-SCALE DRY BALL MILL GRINDING ITABIRITE USING BATCH GRINDING DATA[1]

Alessandro Luiz de Oliveira[2]
Luís Marcelo Marques Tavares[3]

Abstract

Batch grinding tests in lab mills have been widely and, to a great extent, successfully used in the design and optimization of large-scale continuous mills using a couple of popular scale-up procedures. Some researchers, however, argue that the size distribution of ore within a continuous mill is not the same as the one that is tested in batch mode in the laboratory, questioning the basis of such approaches. Indeed, in spite of their widespread use, data are surprisingly scarce that demonstrate the fidelity of the methods that use batch grinding data to predict continuous mill performance, in particular when considering Brazilian iron ores. The paper compares predictions of a continuous pilot-scale mill using the population balance model and the Austin and co-workers scale-up procedure to data from grinding of an Itabirite iron ore and a spent catalyst used in the oil industry. It shows that the deviations encountered between measured and calculated 80% passing sizes for the mill operating under a variety of grinding conditions were, on the average, 7.5 µm, demonstrating the validity of the method. The benefit, however limited, of using an improved prediction of mill hold up of solids is also demonstrated.

Key Words: Modeling; Population balance; Simulation; Grinding.

SIMULAÇÃO DE UM MOINHO PILOTO A SECO NA MOAGEM DE ITABIRITO USANDO DADOS DE MOAGEM DESCONTÍNUA

Resumo

Ensaios de moagem em batelada em moinhos de laboratório têm sido utilizados amplamente e, em grande parte, com sucesso tanto no projeto quanto na otimização de moinhos contínuos, usando alguns procedimentos populares de escalonamento. Alguns pesquisadores, no entanto, afirmam que a distribuição granulométrica do minério dentro de um moinho contínuo não é a mesma que aquela testada nos moinhos em operação em batelada, contestando a base de tais abordagens. De fato, apesar do uso disseminado de métodos de escalonamento que usam os dados da moagem em batelada para prever o desempenho de moinhos contínuos, dados são surpreendentemente escassos que demonstram a fidelidade dessas previsões, especialmente considerando minérios de ferro brasileiros. O trabalho compara as simulações da moagem de um moinho piloto contínuo, usando o modelo do balanço populacional e o método de escalonamento proposto por Austin e colaboradores para dados de moagem de um minério de ferro Itabirítico e de um catalisador gasto. Os resultados mostram que os desvios encontrados entre os valores medidos e calculados para o tamanho 80% passante para o moinho operando sob uma variedade de condições de moagem foram, em média, iguais a 7,5 µm, demonstrando a eficácia do método. O benefício, ainda que limitado, do uso de previsões mais precisas da massa de *hold up* de minério no moinho, é demonstrado.

Palavras-chave: Modelagem; Modelo do balance populacional; Simulação; Moagem.

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[1] *Technical contribution to the 6th International Congress on the Science and Technology of Ironmaking – ICSTI, 42nd International Meeting on Ironmaking and 13th International Symposium on Iron Ore, October 14th to 18th, 2012, Rio de Janeiro, RJ, Brazil.*

[2] *Materials Engineer, M.Sc. candidate, PEMM-COPPE, UFRJ, RJ, Brazil.*

[3] *Mining Engineer, M.Sc., Ph.D., Associate Professor, Programa de Engenharia Metalúrgica e de Materiais, COPPE, UFRJ, RJ, Brazil.*

1 INTRODUCTION

The population balance model (PBM) was first applied 40 years ago to milling as an alternative to the Bond method, which lumps all parameters that influence grinding in both a mill and a classifier operating in closed circuit into a single parameter, the work index. The PBM first had the merit of decoupling the mill and the classifier, and then of accounting for the different functions that describe breakage, internal classification and transport within ball mills.(1) As natural consequences of this approach, a number of researchers have proposed scale-up approaches that rely on batch grinding tests from which functions that characterize the ore-dependent breakage response can be estimated.

The present work compares predictions of a continuous pilot-scale mill using the population balance model and the scale-up procedure proposed by Austin and co-workers to data from grinding both an Itabirite iron ore and ECAT, a spent catalyst from petroleum processing.

2 MATERIALS AND METHODS

2.1 Materials

Samples used in the tests included an Itabirite iron ore from the Iron Quadrangle (Minas Gerais) and ECAT, a spent catalyst used in oil cracking from a plant in Rio de Janeiro. A summary of the samples characteristics is presented in Table 1. The feed size of the Itabirite ore is consistent with that of a typical concentrate for pellet feed production.

The angle of repose of the samples was measured as a means of accounting for the widely different flowabilities of the materials analyzes, being measured using a procedure described elsewhere.(2)

Table 1. Summary of the samples characteristics

Sample	Specific gravity ρ_s (g/cm^3)	F80 (µm)	% - 10 µm	Angle of repose (°)
Itabirite	3.64	177	8.0	31.6
ECAT	2.47	117	0.8	18.6

2.2 Grinding Tests

The mill used in both batch and continuous tests is illustrated in Figure 1. The mill has 31 cm of diameter (*D*) and 32 cm of length (*L*). In batch mode, both the feed and discharge ends were blinded, so as to prevent the discharge of material while running the mill. Tests were then conducted for a number of grinding times, all using a constant steel ball size (15 +/- 2 mm).

In continuous tests, the feed was introduced using a screw vibratory feeder, by Vibrascrew® Inc, at a constant rate and, when steady-state conditions were reached, samples of the mill discharge were collected for analyzes. The mill is equipped with a grate discharge, with 8% open area, with circular openings of 7 mm of diameter. In order to allow the accurate measurement of the discharge rate of solids leaving the mills, a precision scale (Gehaka BG 8000) was positioned below the discharge trough. The attainment of steady-state conditions in the continuous ball milling tests was verified by both comparing feed and discharge rates, and analyzing the stability in the mill hold-up value, measured using load cells.

ISSN 2176-3135

After all tests size analyzes were conducted by laser scattering using a Malvern Mastersizer® 2000 of the samples dispersed in water.

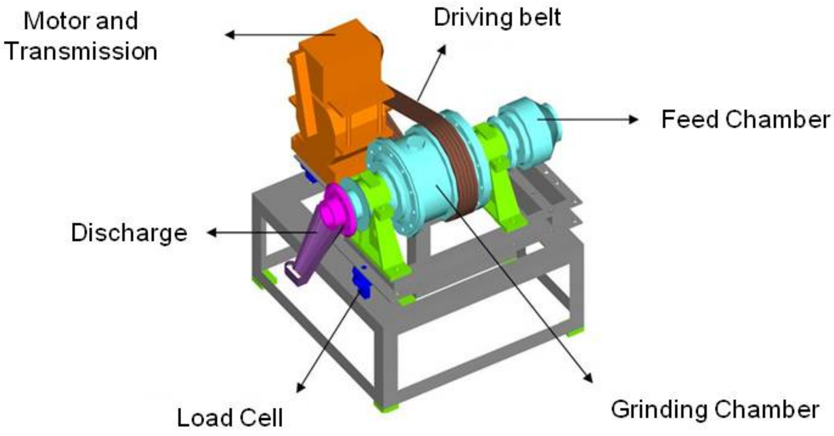

Figure 1. Schematic diagram of the mill.

3 RESULTS AND DISCUSSION

3.1 Batch Grinding

Grinding can be described using the size-mass balance model which, for a batch grinding process, is given by Austin, Klimpel and Luckie. (1)

$$\frac{dw_i}{dt} = -S_i w_i + \sum_{j=1}^{i-1} S_j b_{ij} w_j \qquad (1)$$

Where w_i is the fraction of particles contained in size interval i, S_i is the specific breakage rate and b_{ij} is the discrete breakage function. By using appropriate values of these functions the entire size distributions generated after different grinding times may be fitted to experimental data.

The specific breakage rate is significantly influenced by particle size, being successfully described by Austin, Klimpel and Luckie. (1)

$$S_i = S_1 \left(\frac{x_i}{x_o} \right)^{\alpha} \qquad (2)$$

Where x_o is the reference size, taken as 1 mm. In the equation, no correction for abnormal breakage of the coarse material was introduced, given the fine size of the feed in the present work.

The breakage distribution function, on the other hand, is well described in its most general form as: (3)

$$B_{ij} = \Phi \left(\frac{x_{i-1}}{x_j} \right)^{\gamma} + (1-\Phi) \left(\frac{x_{i-1}}{x_j} \right)^{\beta}$$

for $x_{i-1} > D^*$(3)

and

$$B_{ij} = \Phi\left(\frac{x_{i-1}}{x_j}\right)^{\gamma}\left(\frac{x_{i-1}}{D^*}\right)^{\omega} + (1-\Phi)\left(\frac{x_{i-1}}{x_j}\right)^{\beta}$$

for $x_{i-1} \le D^*$

(4)

The breakage distribution function in density form is simply computed from

$$b_{ij} = B(x_i;x_j) - B(x_{i+1};x_j)$$
$$b_{ii} = 0 \qquad (5)$$

Parameters in Equations (2-4) have been fitted to batch grinding data of the Itabirite iron ore and ECAT in the mill depicted in Figure 1 (Table 2), and results are shown in Figure 2, which demonstrates the very good fit to the data. The relationship between the solids hold up in the mill (*M*) and the interstitial filling (*U*) is given by Austin and Tangsathitkulchai(4)

$$M = 0.24\,\pi\,D^2\,L\,J\,U\,\rho_s\,/\,4$$

(6)

Where *D* is the mill diameter, *L* the length, *J* the ball filling and ρ_s is the solids density.

Table 2. Summary of breakage characteristics of the materials studied

Sample	Breakage rate function		Breakage distribution function				
	S_1 (min^{-1})	α	Φ	β	γ	ω	D^* (µm)
Itabirite	2.95	1.90	1.00	0.04	0.56	0.49	5.30
ECAT	11.27	1.66	1.00	0.99	0.60	1.11	2.96

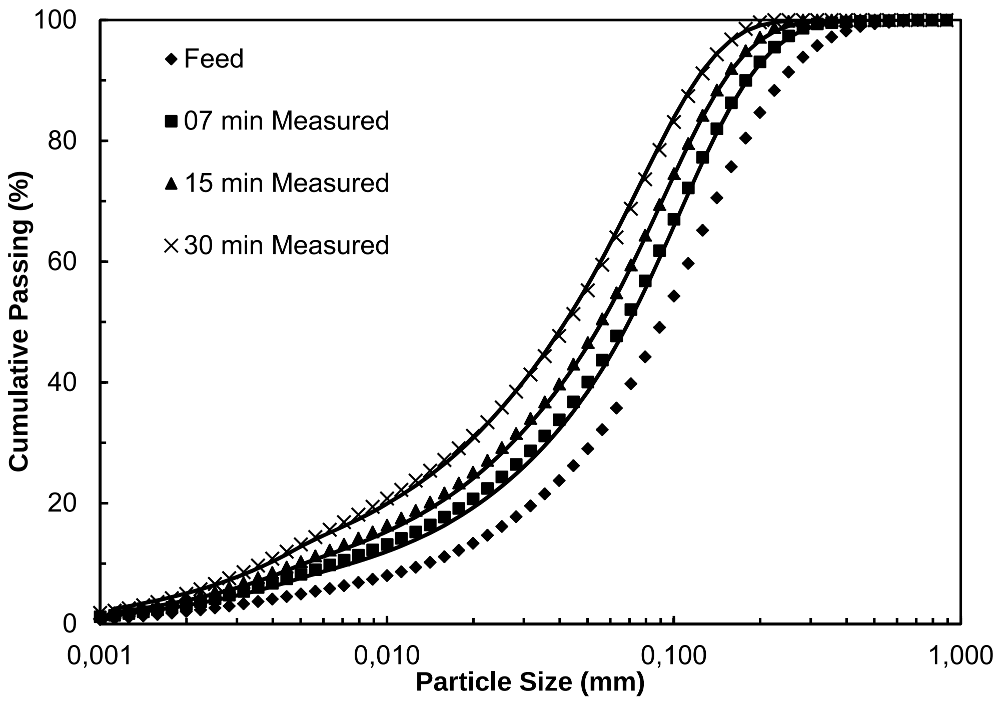

Figure 2. Comparison between measured and fitted size distributions from batch grinding of Itabirite iron ore (J = 0.30, U = 1.04 and φ_c = 0.75): symbols are experimental data and lines represent the model fit.

From these, the model can be used to predict the size distributions from grinding, departing from a feed containing a given size distribution. However, in order to do that, it is necessary to describe the pattern of mixing of solids within the mill. The residence time distribution for the mill in question has been measured (5) and Figure 3 shows a comparison between measured results and fitting using a model that is based on three mixers in series, given by Oliveira, Carvalho e Tavares.(5)

$$E(t)=\frac{\theta_1(\theta_2-\theta_3)\exp(-t/\theta_1)+\theta_2(\theta_3-\theta_1)\exp(-t/\theta_2)+\theta_3(\theta_1-\theta_2)\exp(-t/\theta_3)}{(\theta_1-\theta_2)(\theta_3-\theta_1)(\theta_2-\theta_3)} \quad (7)$$

Where θ_1, θ_2 and θ_3 are the mean residence times in the three mixers. Given as a function of the total residence time within the mill (θ), the values were $\theta_1/\theta = 0.1$, $\theta_2/\theta = 0.83$, $\theta_3/\theta = 0.07$.

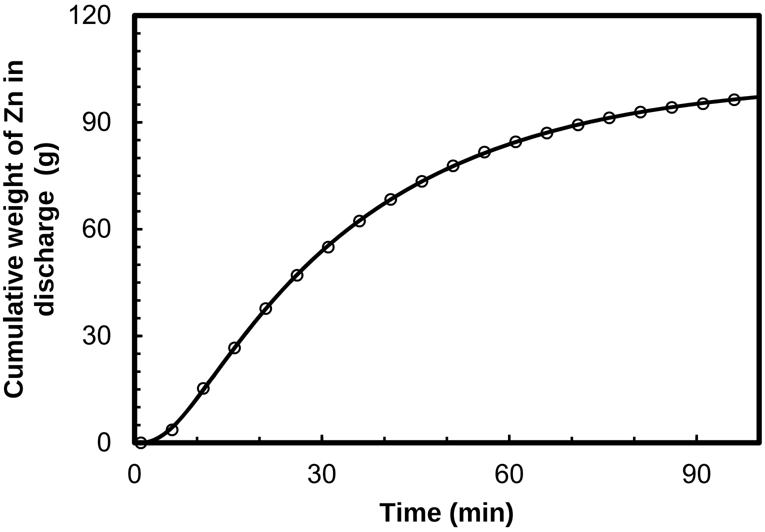

Figure 3. Cumulative residence time distribution of solids within the mill from Figure 1 (5): symbols represent experimental data and the line represents the fit to the model corresponding to three mixers in series (Equation 7)

Consistent with this model, it is now possible to describe grinding in three continuous perfectly mixed regions operating under steady-state conditions as

$$w_i^{(1)}=w_i^F-S_i\,w_i^{(1)}\,\frac{0.1M}{W}+\sum_{j=1}^{i-1}b_{ij}S_jw_j^{(1)}\,\frac{0.1M}{W} \quad (8)$$

$$w_i^{(2)}=w_i^{(1)}-S_i\,w_i^{(2)}\,\frac{0.83\,M}{W}+\sum_{j=1}^{i-1}b_{ij}S_jw_j^{(2)}\,\frac{0.83\,M}{W} \quad (9)$$

$$w_i=w_i^{(2)}-S_i\,w_i\,\frac{0.07\,M}{W}+\sum_{j=1}^{i-1}b_{ij}S_jw_j\,\frac{0.07\,M}{W} \quad (10)$$

Where w_i^F is the feed size distribution, w_i is the size distribution of the product leaving the mill, whereas $w_i^{(1)}$ and $w_i^{(2)}$ are the size distributions of the material leaving mixing zones 1 and 2, respectively. W is the feed flow rate of solids and M is the solids hold-up within the mill. Equations (8-10) can be solved recursively and sequentially, and a Matlab® code was written for that purpose. It is important to note that no exit classification was introduced in the model. This is justified by the large grate openings present in the mill and the fact that no forced suction of air (air sweeping) was used during the tests.

Unfortunately, the PBM gives no information *a priori* that allows predicting how these functions, in particular the breakage rates, are affected by milling conditions, that would make it usable for simulating the continuous mill under conditions that differ from those tested in the batch mill. In order to overcome this limitation, empirical expressions have been proposed for this purpose.(1,6) The approach used in the present work is the one proposed by Austin and co-workers. It is essentially based on the empirical scale-up of the breakage rate function, using the relationship (1)

$$\frac{S_1}{S_{1b}}=\left(\frac{1+6.6J_b^{2.3}}{1+6.6J^{2.3}}\right)\left(\frac{\psi_c-0.1}{\psi_{cb}-0.1}\right)\left(\frac{1+\exp[15.7(\varphi_{cb}-0.94)]}{1+\exp[15.7(\varphi_c-0.94)]}\right)\exp[1-1.32(U-U_b)] \qquad (11)$$

Where subscript *b* corresponds to experimental conditions used in the batch test mill. Since the same mill diameter is used both in the batch and continuous grinding tests, then the mill size correction has been removed from the equation.

One potential challenge identified in the application of the scale-up procedure by Austin and co-workers is the prediction of the mill hold-up of solids (*M*) for a given set of milling conditions and feed rate (*W*). Although this was actually measured directly in the tests, it is initially assumed in the simulations that it was maintained constant at *U* = 1, leading to values of M that are estimated using Equation 6. As such, the continuous mill was simulated and Figure 4 compares measured to calculated size distributions from the continuous mill operating under a range of feed sizes, showing the good agreement between the two, considering that the model was not fitted to this data. A large number of tests, covering a range of values of *J* and φ_c has been conducted with the two materials studied, and Figure 5 compares the P80 (80% passing size) values from experiments and simulations. It is evident that the general agreement is good, with an average standard error of 7.5 µm in the predictions.

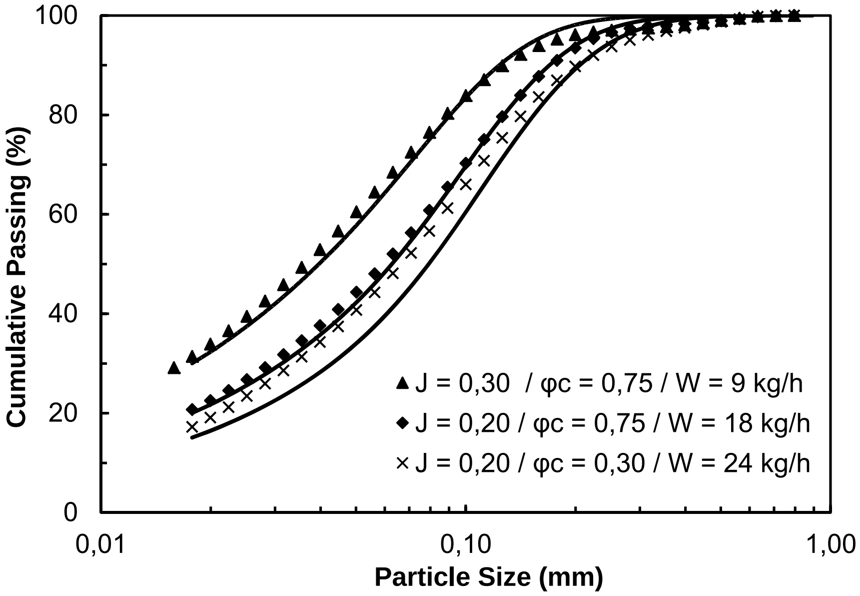

Figure 4. Comparison of measured (symbols) and predicted (lines) particle size distributions from continuous grinding tests of Itabirite iron ore: symbols represent experimental data and lines model predictions.

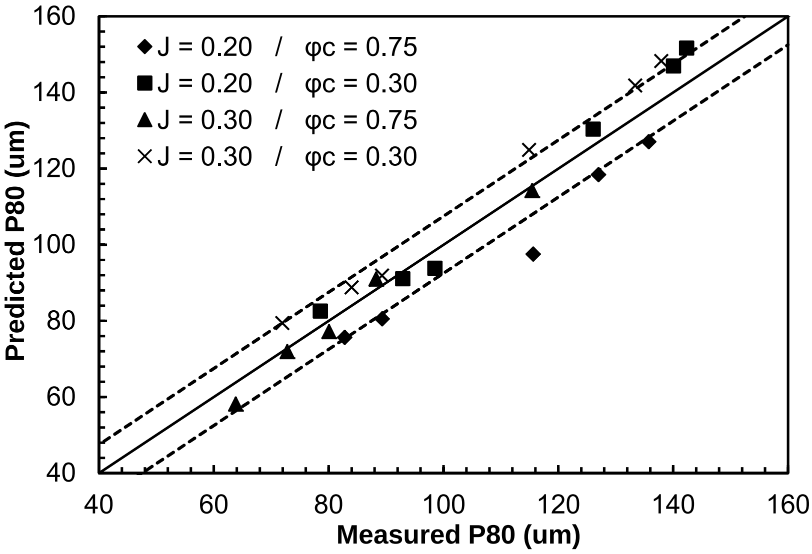

Figure 5. Comparison between measured and predicted P80 values for tests for the two materials studied, assuming *U* = 1. Dashed lines represent the standard deviations of the predictions.

Finally, Figure 6 shows the variation of interstitial filling (*U*) as a function of grinding conditions, including feed rate. It shows that values obtained for iron ore are, on the average, higher than those found for ECAT, and also the values differ significantly from the assumed value of *U* = 1 consider in the previous simulations. As such, simulations of batch grinding were then conducted considering the actual values of mill hold up and comparisons are shown in Figure 7 between measured and simulated P80 values. It is evident that deviations between actual and simulated values are reduced if the actual hold-up values are known. Indeed, these errors reduce to 6.7 µm in that case. Work is ongoing in the authors´ laboratory to develop an appropriate model that will be able to describe that.

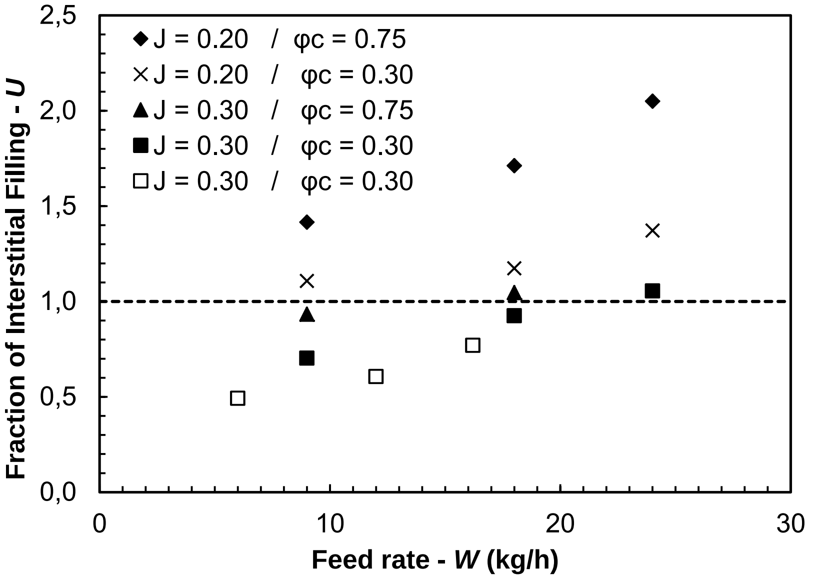

Figure 6. Interstitial filling (*U*) values measured in continuous tests for iron ore and ECAT as a function of feed rate.

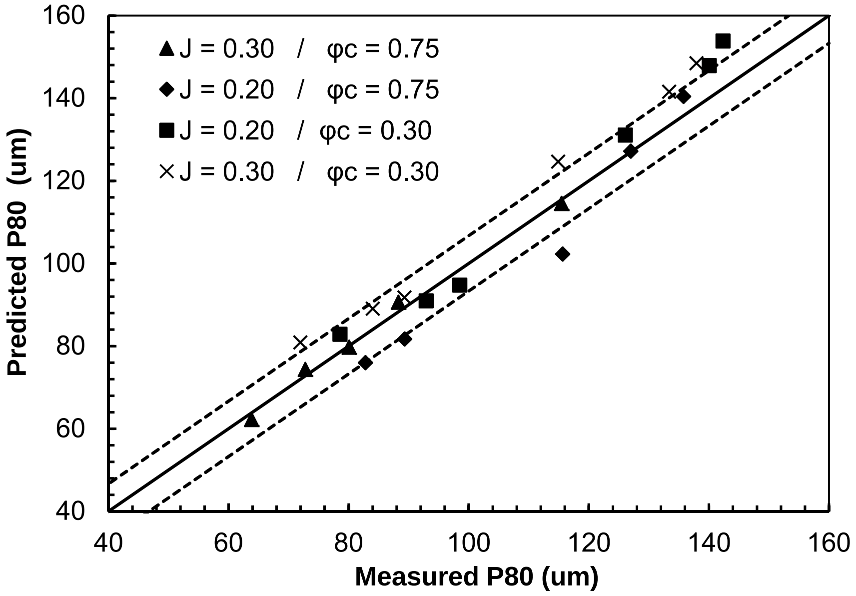

Figure 7. Comparison between measured and predicted P80 values for tests conducted using the two materials studied (*U* calculated from measured *M*). Dashed lines represent the 6.7 µm error band.

4 CONCLUSIONS

The size-mass balance model has been successfully used to describe batch grinding data of an Itabirite iron ore and a spent catalyst.

Considering the same mill, now operating in continuous mode, as three mixers in series, experiments and simulations have been compared for a range of fractions of critical speeds, fillings and feed rates with good agreement, but with deviations as high as 9.6 µm between measured and calculated P80s. It is demonstrated that improved predictions can be achieved if accurate estimates of the mill interstitial filling (*U*) are available as a function of operating characteristics.

Acknowledgements

The authors would like to acknowledge the support of CNPq and CAPES to this investigation. Also, the assistance of personnel from the Laboratório de Tecnologia Mineral staff, that is, Eng. Rodrigo M. de Carvalho in the development of the computer codes, Eng. Yemcy Flores in the Malvern analyzes and Evandro Ramos in the angle of repose measurements is deeply appreciated.

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