STRESS-STRAIN CURVES FOR STEEL FIBER-REINFORCED CONCRETE IN COMPRESSION[1]

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Abstract
This paper presents a study on the compressive behavior of steel fiber-reinforced concrete. In this study, an analytical model for stress-strain curve for steel fiber-reinforced concrete is derived for concretes with strengths of 40 MPa and 60 MPa at the age of 28 days. Those concretes were reinforced with steel fibers with hooked ends 35 mm long and with aspect ratio of 65. The analytical model was compared with some experimental stress-strain curves and with some models reported in technical literature. Also, the accuracy of the proposed stress-strain curve was evaluated by comparison of the area under stress-strain curve. The results showed good agreement between analytical and experimental data and the benefits of the using of fibers in the compressive behavior of concrete.

Key-words: steel fiber-reinforced concrete, stress-strain curves, compressive behavior.

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1 INTRODUCTION

The capacity of plain concrete to absorb energy as it deforms may not be sufficient when the structure constituted by this material must support large displacements. In this sense, the addition of steel fibers to the matrix increases, beyond the capacity of energy dissipation, the tensile, fatigue and impact strengths, the toughness and the fracture energy of the material. These improvements make the steel-fiber reinforced concrete a composite material very interesting for structural applications.

When designing or analyzing a structure, the structure strength can be determined if the materials strengths are known. Thus, the structure is safe for certain efforts since these efforts do not produce stresses exceeding the material strength. However, the fact that structure is safe for design efforts does not prevent a possible brittle failure. So, to be considered safe, the structure must also provide ductility, which can be achieved using tougher materials such as steel fiber-reinforced concrete to build structural elements.

To verify if the material presents toughness compatible with the level of ductility that must be provided to the structure, the stress-strain curves can be used. So, toughness is related to the area under this curve and, the greater the area, the greater the material toughness.

With the increasing development of engineering and the increasing on complexity of geometry and loads used in the projects, design and analysis of concrete structures have also become more complex. Thus, the use of numerical methods, such as finite element method, has become common practice in analysis of these structures. From this point of view, the stress-strain curves, representing constitutive models for material, become even more important since the accuracy of analysis depends on the ability of constitutive models to represent properly the material behavior.

In technical literature there are reported many analytical models developed to represent the stress-strain curves for plain concrete under compression. Among most important and known models, must be cited models Popovics (1973) and Carreira and Chu (1985). Since the models of the compressive behavior of fiber-reinforced concrete, in general, were developed from models developed for plain concrete, it is necessary the inclusion of some parameters in these models to consider the influence of fibers on the properties of stress-strain curve. The models of Ezeldin and Balaguru (1992), Mansur *et al*. (1999), Nataraja *et al*. (1999), Barros and Figueiras (1999) and Araújo (2002) can be cited as examples.

The model proposed by Ezeldin and Balaguru (1992) is valid for concretes with compressive strength varying from 35 MPa to 85 MPa. Steel fibers with hooked ends and with aspect ratio of 60, 75 and 100 were added in volumetric fractions of up to 0.75%. The model proposed by Mansur *et al*. (1999) was developed for concrete with compressive strength varying from 70 MPa to 120 MPa to which were added up to 1.5% of steel fibers with hooked ands and aspect ratio of 60. Nataraja *et al*. (1999) proposed a model valid for concretes with compressive strength varying from 30 MPa to 50 MPa using up to 1.0% of crimped steel fibers with aspect ratio of 55 and 82. Beyond these studies, Barros and Figueiras (1999) presented a model developed for concretes with compressive strength varying from 30 MPa to 60 MPa, to which were added up to 0.75% of steel fiber with hooked ends and aspect ratios of 60 and 75. Finally, the model presented by Araújo (2002) is valid for concretes with compressive strength varying from 50 MPa and 100 MPa, to which were added up to 2.0% of steel fiber with hooked ends, 30 mm long and with aspect ratio of 48.

All these models, except the one proposed by Barros and Figueiras (1999), are based on the model proposed by Carreira and Chu (1985) (or on models derived from this), whose general expression is given by equation (1), in which σ_c the compressive stress, f_c is the compressive strength, ε_c is strain, ε_{c0} is the peak strain and β is the factor which considers the influence of fibers on the curve form. The parameters β and $\varepsilon_{c,0}$ can be obtained, in general, by equations that correlates these parameters to fiber volumetric fraction and/or to compressive strength of concrete and, obviously, are different from one model to another.

$$\frac{\sigma_c}{f_c} = \frac{\beta\left(\dfrac{\varepsilon_c}{\varepsilon_{c,0}}\right)}{\beta - 1 + \left(\dfrac{\varepsilon_c}{\varepsilon_{c,0}}\right)^{\beta}} \qquad (1)$$

In this paper, results of many compression tests with displacement control performed by the authors were used. Concretes presented average compressive strength of 40 MPa and 60 MPa and were produced with steel fibers 35 mm long. These fibers had aspect ratio of 64 and were added in fiber volumetric fractions of 1.0% and 2.0%. The influence of fibers was evaluated on the peak stress and strain and on toughness in compression. An analytical model to get the complete stress-strain curve was developed based on the model of Carreira and Chu (1999).

2 MATERIAL E METHODS

2.1 Composition of concretes

Concretes with 40 MPa were produced with Portland cement type CP II F 32 (cement with limestone filler), sand, coarse aggregate with maximum size of 12.5 mm, water and superplasticizer admixture. To improve the workability of fresh concrete and reduce the consumption of cement, 5% of silica fume were added in partial replacement of cement and 20% of fly-ash as mineral addition. To these concretes steel fibers 35 mm long were added in fiber volumetric fractions of 1.0% and 2.0%, having been the fiber aspect ratio of 64. Concretes with 60 MPa were produced with the same materials, except fly-ash.

Table 1 presents compositions of the produced concretes. It was observed that amount of superplasticizer used in each mixture increased as fiber volumetric fractions increased, since the difficulty in adding high amounts of fibers to concrete increases as the amount of fibers increases. Also, the superplasticizer dosage and the water-cement ratio were calculated in relation to equivalent cement, that is, in relation to a mixture of cementitious materials (cement, silica fume and fly-ash) with the same density of cement.

Three cylindrical specimens 150 mm diameter and 300 mm high were cast for each mixture. All specimens were stored in a humid chamber, in which temperature and humidity were kept at approximately 23ºC and 95%, respectively, until testing.

Table 1. Composition of studied mixtures (kg/m^3).

Material	Fiber volumetric fractions, Vf (%)											
	0.0%				1.0%				2.0%			
	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12
Equiv. Cem.*	442	458	456	460	421	458	457	457	429	439	435	461
Cement	265	435	433	437	316	435	434	434	322	417	413	438
Silica fume	32	16	16	17	15	16	16	16	15	16	16	17
Fly-ash	102	-	-	-	63	-	-	-	64	-	-	-
Sand	884	994	988	997	773	993	992	991	788	952	944	999
Coarse aggreg.	884	797	793	800	902	797	794	795	919	763	757	801
Water	172	164	163	165	198	164	164	164	202	207	205	165
Superplasticizer	3.27	8.25	5.47	8.27	4.21	9.16	9.15	6.85	6.44	8.77	10.87	12.00
$a/c_{equiv.}$	0.39	0.36	0.36	0.36	0.47	0.36	0.36	0.36	0.47	0.36	0.36	0.36
f_{cm} (MPa)	40.30	49.28	53.37	52.40	40.42	62.59	66.17	71.70	36.73	58.95	60.91	66.41

*Equivalent cement is a mixture of cementitious materials with the same density of cement.

2.2 Methodology

The produced specimens were subjected to compression tests with displacement control for obtaining stress-strain curves in compression. In this test, displacements were limited to measurement capacity of the transducers that was 10 mm. However, this limitation applies only to the maximum displacement, although it is possible that some specimens present smaller displacements at the end of the test.

Once stress-strain curves are determined, the average curves were obtained for each produced mixture. Following, these curves were normalized dividing the stresses axis by the peak stress. This procedure removes the influence of compressive strength and allows direct comparison of curves. The strain axis was also normalized, but by peak strain, once the model proposed by Carreira and Chu (1985) uses normalized strains.

In this work, the equation proposed by Carreira and Chu (1985) was chosen to perform nonlinear regressions of curves and to obtain the value of β, since the form of this curve is similar to experimental curves. Also, an equation for estimating peak strain by means of nonlinear regressions of this property with compressive strength and fiber volumetric fraction was proposed.

The model was validated by means of assessing of ratio between experimental and analytical values of relative toughness. Obviously, ratios around 1.0 are ideal, but a difference of ±10% between experimental and analytical results was considered satisfactory.

3 RESULTS AND DISCUSSION

The model proposed by Carreira and Chu (1985) was used in non-linear regressions to determine the β values, which were correlated with the fiber volumetric fraction and with the compressive strength of concrete, as shown in equation (2). The correlation is shown in Figure 1. It can be observed from this figure a trend to increasing of β values as the compressive strength of concrete increases. However, the opposed trend is verified with the addition of fibers to concrete, being the lowest β values obtained for higher amounts of fibers.

$$\beta = \left(0.0536 - 0.5754 V_f\right) f_c \tag{2}$$

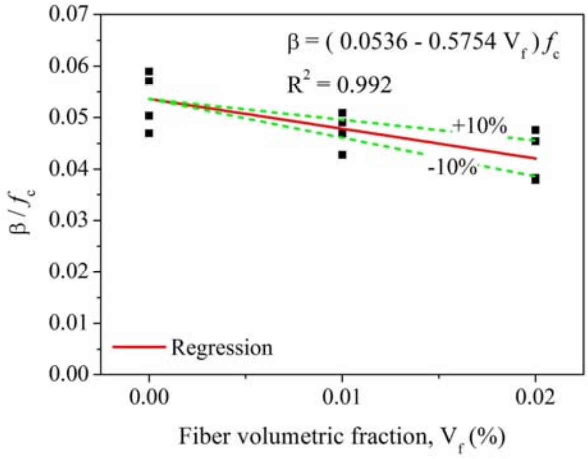

Figure 1. Regression of β.

As the proposed model depends on the peak strain to define the complete stress-strain curve of the material, it was determined equation (3) which relates the peak strain to the concrete compressive strength and to the fiber volumetric fraction. Regressions performed to obtain this equation are shown in Figure 2, as well as the confidence intervals for a significance level of 90%.

$$\varepsilon_{c,0} = \left(0.00048 + 0.01886\, V_f\right) \ln f_c \tag{3}$$

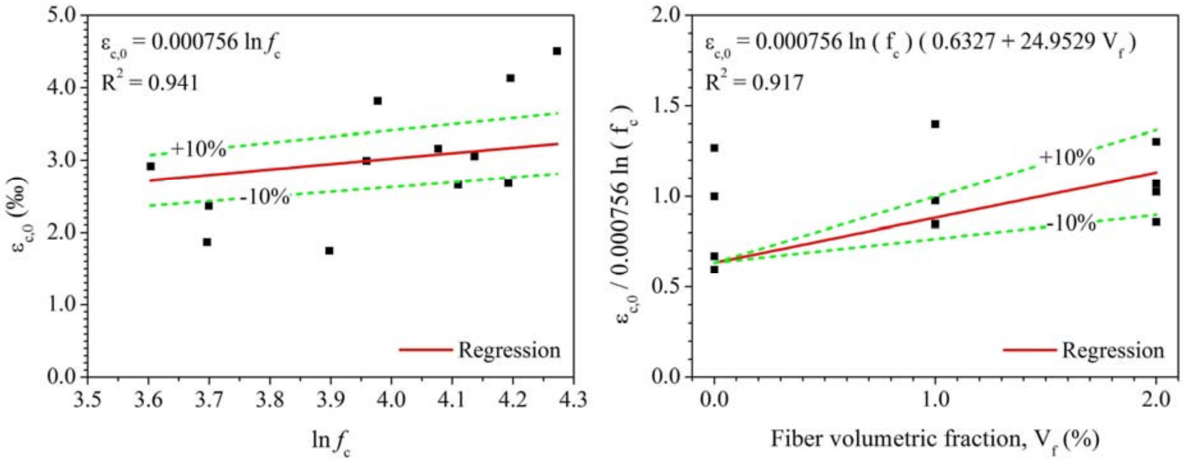

Figure 2. Regressions to define equation to estimate the peak strain.

Once the β value is known, the equation that describes the proposed model gets defined by replacement of this parameter in equation (1). So, the fiber volumetric fraction, the peak strain and the concrete compressive strength must be provided to define the complete stress-strain curve of steel fiber-reinforced concrete in

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compression. Replacing equation (2) in equation (1) and giving suitable values to parameters of the model, analytical curves can be obtained and after compared to experimental curves shown in Figure 3.

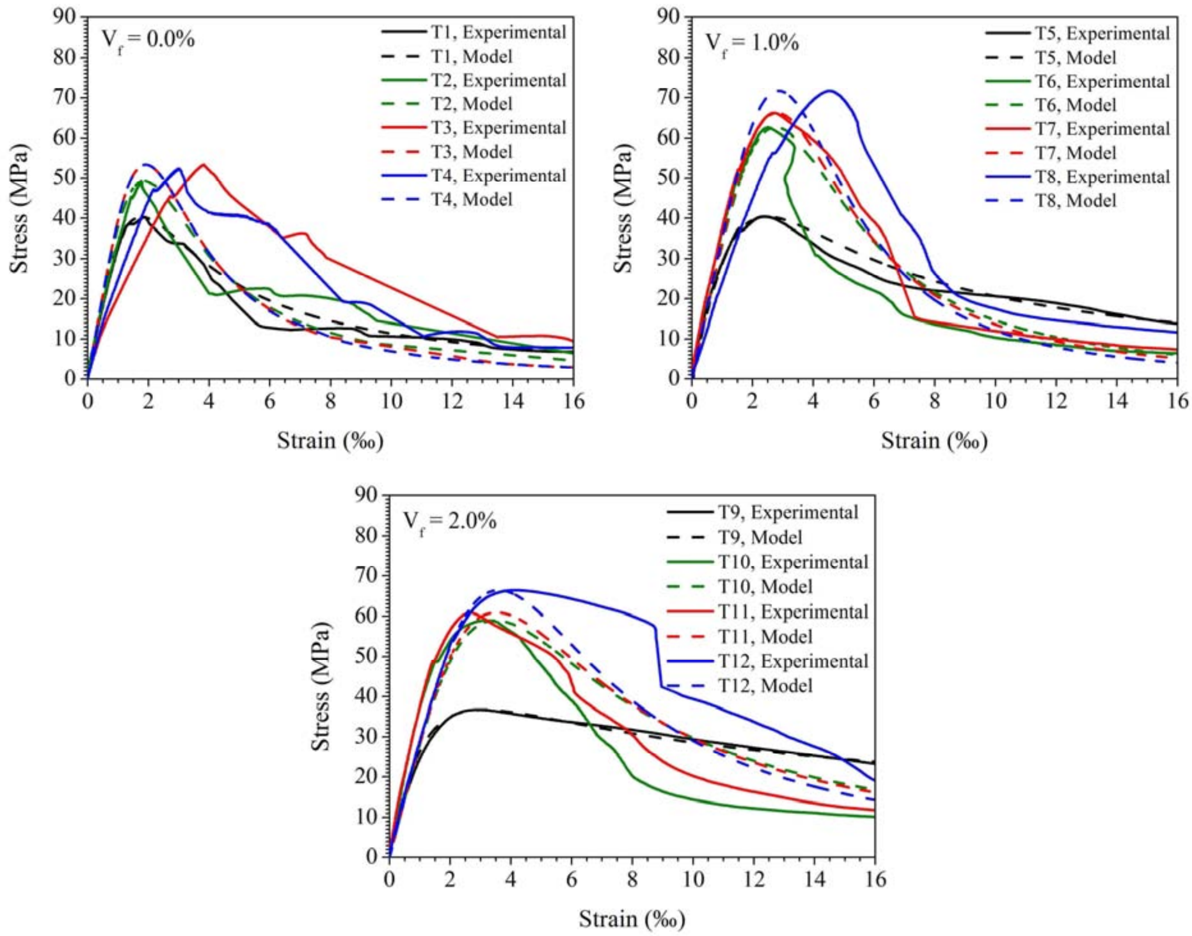

Figure 3. Comparison between experimental and analytical results.

It can be observed from this figure, in general, that results obtained by proposed model are well fitted to experimental results. Also, in the most cases, the material behavior in elastic range was accurately represented, except for the mixture T8. This was expected, since the action of fibers in elastic range can be neglected and the equation used in the non-linear regressions was originally developed for plain concrete. Also, the shape of analytical curve was reasonably similar to experimental curve shape, except in some cases, which presented significantly differences.

Figure 4 shows a brief analysis to evaluate the influence of compressive strength and fiber volumetric fraction on the stress-strain curve shape and in its properties. It can be observed in this figure that increasing on the concrete compressive strength makes stress-strain curve less stretched and increases the peak stress. As it was expected, for the same compressive strength, the increasing on the fiber volumetric fraction is associated to increasing on toughness.

Until now, only qualitative evaluations were done to verify the validity of proposed model. However, quantitative evaluations are needed to demonstrate the accuracy of proposed model in estimating of properties of stress-strain curve, namely peak strain and toughness. In this sense, some comparisons between energy obtained by

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proposed model and by experimental results were done, allowing quantifying the differences between the shape of experimental and analytical curves. So, the concept of relative toughness was applied, which relates the energy dissipated by the material and the energy dissipated by a perfectly plastic material up to maximum strain of 15‰. Similar comparison was done for peak strain. Table 2 shows the results obtained from these comparisons.

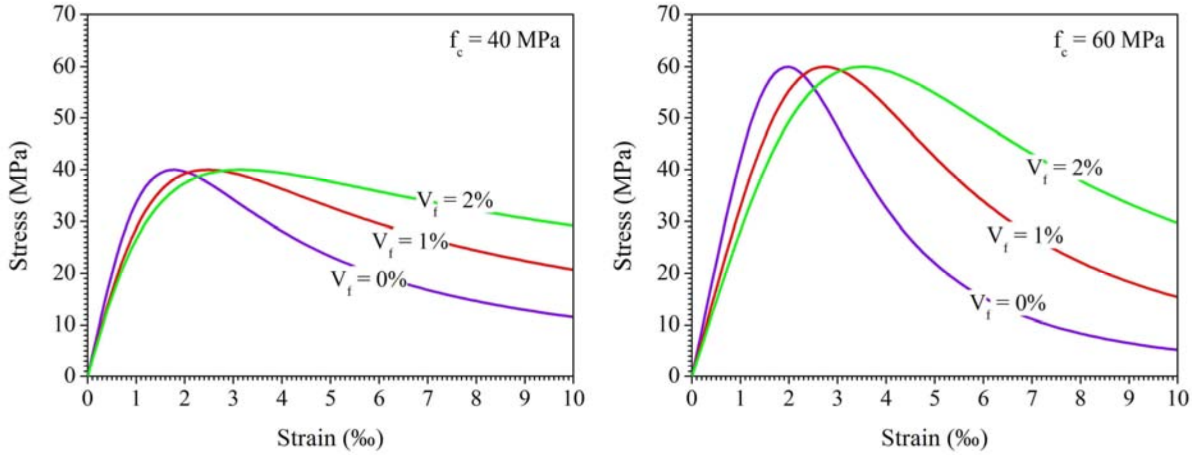

Figure 4. Analytical curves for concretes of 40 MPa and 60 MPa.

From this table, variations of -21% (for mixture T10) and +52% (for mixture T4) in the ratio between relative toughness values obtained from experimental results and from proposed models were observed. The peak strain also presented high variations: -24% (for mixture T11) to +100% (for mixture T3). These differences between experimental and analytical values are due to the experimental curve shape on descending branch, whose behavior is affected by the high scattering intrinsic to the adding of fibers to concrete, since ascending branch was represented satisfactorily in the most cases. Values of relative toughness and peak strain with reasonable accuracy and confidence level of 90% were observed in 50% of the cases. Although the differences observed between experimental and analytical results, the model can be considered as valid.

Table 2. Validation of proposed model.

Traço	f_c (MPa)	A_{exp}	A_{theo}	RT_{exp}	RT_{theo}	RT_{exp} / RT_{theo}	$\varepsilon_{c,0,exp}$	$\varepsilon_{c,0,theo}$	$\varepsilon_{c,0,exp}$ / $\varepsilon_{c,0,theo}$
T1	40.30	0.253	0.278	0.419	0.460	0.91	0.00187	0.00177	1.05
T2	49.28	0.302	0.278	0.409	0.376	1.09	0.00175	0.00187	0.94
T3	53.37	0.418	0.275	0.522	0.344	1.52	0.00382	0.00191	2.00
T4	52.40	0.374	0.271	0.476	0.345	1.38	0.00299	0.00190	1.57
T5	40.42	0.367	0.381	0.606	0.629	0.96	0.00237	0.00247	0.96
T6	62.59	0.325	0.412	0.346	0.439	0.79	0.00306	0.00277	1.10
T7	66.17	0.420	0.416	0.423	0.419	1.01	0.00268	0.00280	0.96
T8	71.70	0.492	0.423	0.457	0.393	1.16	0.00451	0.00286	1.58
T9	36.73	0.447	0.445	0.812	0.808	1.01	0.00292	0.00309	0.94
T10	58.95	0.431	0.543	0.488	0.614	0.79	0.00316	0.00349	0.90
T11	60.91	0.489	0.550	0.535	0.602	0.89	0.00266	0.00352	0.76
T12	66.41	0.694	0.568	0.697	0.570	1.22	0.00414	0.00360	1.15

A: area under stress-strain curve; RT: relative toughness; exp: experimental; theo: theoretical

4 CONCLUSIONS

Based on obtained results and on presented analysis, it can be concluded the following:

- The proposed model allow to estimate the complete stress-strain curve for plain concrete and concretes reinforced with up to 2% of steel fibers with hooked ends, since this concretes have compressive strength between 40 MPa and 60 MPa;
- The differences between experimental and analytical results were acceptable and are due to the shape of descending branch of experimental curve, in which irregularities that does not occur in other models available in technical literature are usually found;
- The proposed model depends only on the fiber volumetric fraction and on concrete compressive strength, which makes easy its using in comparison with other models which depend on more parameters.

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