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Theme: Ore Mining and Treatment

INTEGRATING DATA OF DIFFERENT PRECISION*

Marcel Antonio Arcari Bassani[1]
Cristina da Paixão Araújo[2]
João Felipe Coimbra Leite Costa[3]
Roberto Quadros Menin[4]

Abstract

In the mining industry, it is common to have data collected in various formats. In an open pit mining operation, at the early stages of exploration samples are obtained from diamond drillholes. During the production stage, samples are collected from blastholes. The last is more numerous but less precise. From a geostatistical view, this dissimilarity has to be considered to integrate the two sources of information. To combine this different data, three methodologies were investigated: simple kriging with local varying mean, cokriging and ordinary kriging. The exhaustive Walker Lake data set was used and it is considered the source to obtain the true grades. Initially, samples were obtained from the exhaustive data set at a regular spacing grid of 20 x 20 meters. Next, samples were obtained again from the exhaustive dataset at a regular spacing of 5 x 5 meters and a relative error of 25% was added. Then, both data were used to estimate blocks using the three methods mentioned. The grade tonnage curves were compared with the true block grade distribution. Moreover, the block misclassification was evaluated. The results showed that ordinary kriging produced estimates closer to the true block grade distribution and reduced the block misclassification.

Keywords: Precision; Misclassification; Sampling.

[1] *Mining Engineer/Mining Engineer, MSc Student, Mining Engineering Department, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.*

[2] *Chemistry/Chemist, MSc Student, Mining Engineering Department, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.*

[3] *Mining Engineer/Phd, Associate Professor, Mining Engineering Department, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.*

[4] *Mining Engineer/Mining Engineer, MSc Student, Mining Engineering Department, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.*

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1 INTRODUCTION

In the mining industry, it is common to have data collected in various formats. During the exploration stage, samples are obtained from diamond drill holes, which have high quality and the sampling error is usually negligible. During the production stage, samples are usually collected from blastholes, which may contain large sampling errors [1,2]. From a geostatistical view, this difference in precision has to be considered to integrate the two data types. The aim of this paper is to investigate three geostatistical methodologies to combine them: cokriging [3], simple kriging with local varying mean [3] and ordinary kriging [4].
The estimates for each scenario were compared with a reference block grade model. As blasthole sampling is mainly used for the short term model, the results emphasize the impact in the block misclassification.

2 MATERIAL AND METHODS

2.1 Presentation of the Data

This study uses the exhaustive Walker Lake dataset [5] with 78000 point support samples distributed regularly at 1x1 meter. The variable V was used study and the original unit was rescaled so that it is now resembles a copper mineral deposit. In order to obtain the reference block grade distribution, the exhaustive point support dataset was averaged into 3210 blocks of size 5 x 5 meters. These blocks represent the true block grades and were used for comparisons.
Two types of data were considered in this study. First, samples were obtained from the exhaustive point support dataset at a regular spacing of 20 x 20 meters. These samples do not have measurement errors and mimick diamond drillhole samples. Second, samples were obtained from the exhaustive point support dataset at a regular spacing of 5 x 5 meters, and a random relative sampling error of 25% was added (or subtracted). The relative error is assumed to follow a Gaussian distribution with zero mean and standard deviation determined by the product of the relative error and the grade [6, 7]. These samples represent the blasthole samples, which have poorer quality than the diamond drillhole data. The error is assumed to be heteroscedastic, which occurs frequently in practice [3, 8]. Table I shows the summary statistics of the reference point support dataset, of the reference block support dataset, of the sample dataset without error and of the sample dataset with 25% of relative error. The two samples datasets have mean very close to the true mean, which indicate that there is no global bias.

Table 1. Summary statistics of the data.

Data	Nº. Samples	Mean	St. Dev	Minimum	Maximum	CV
Reference Point Support	78 000	2.78	2.50	0.00	16.31	0.90
Reference Block Support	3120	2.78	2.29	0.00	13.78	0.82
V samples with error	3120	2.79	2.70	0.00	0.00	0.97
V samples without error	195	280	2.43	0.00	10.74	0.87

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2.2 Estimation Methodologies

Three methodologies were evaluated for the estimation of block grades: simple kriging with local varying mean [3], ordinary cokriging [3] and ordinary kriging [4].

2.2.1 The Simple Kriging with Local Varying Mean

Simple kriging with local varying mean estimator [4] is defined by Equation (1):

$$Z^*(u) - m^*(u) = \sum_{i=1}^{n} \lambda(u) \cdot [Z(u_\alpha) - m^*(u_\alpha)] \qquad (1)$$

Where,

- Z*(u) is the estimate at location u.
- Z(uα) is the sample value at location uα.
- m*(u) is the local mean of the attribute at location u.
- m*(uα) is the local mean of the attribute at location uα.
- λ(u) is the weight associated to the sample at location uα for the estimation at location u.

The local varying mean of the variable V (m*(u) in Equation 1) was defined using a linear regression between the samples without error and the samples with error. The linear relation was defined using the method of the least square. Figure 1 shows the scatter plot between V samples without error and V samples with 25% of relative error. The picture also shows the equation of the linear regression. There is a poor correlation between the two samples and the true values scatter significantly from the line fitted.

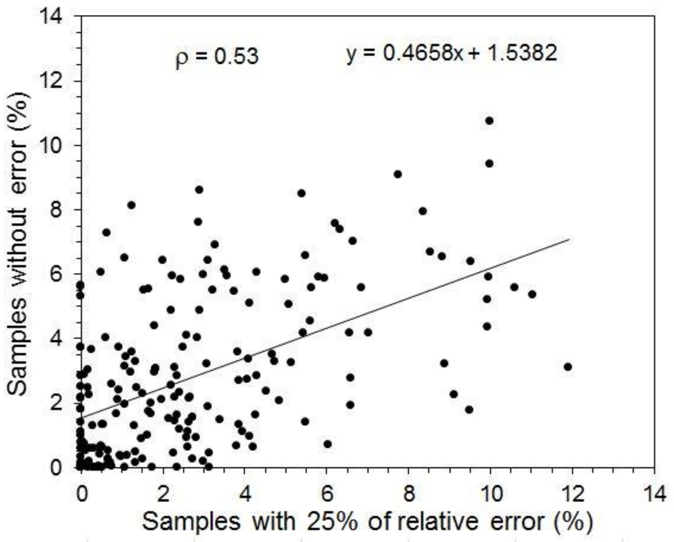

Figure 1. Scatter plot between samples without error and samples with 25% of relative error and linear regression.

The residual variogram used in the simple kriging with local varying mean was defined by Equation (2):

$$\gamma_r(h) = 1.8 + 2.44 \cdot Sph(1)\left(\frac{N157.5E}{53m}, \frac{N67.5E}{20m}\right) \qquad (2)$$

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2.2.2 Ordinary Cokriging

The Ordinary Cokriging is thoroughly explained in Goovaerts [3]. This is the classic methodology to incorporate data of different quality. It takes into consideration the auto and cross spatial correlation among the variables involved. In order to avoid negative weights given to the secondary variable, the standardized ordinary cokriging was used [3, 5]. The spatial continuity was defined using the Linear Model of Coregionalization, which is defined by equations (3), (4) and (5). The primary variable is the V sample without error whereas the secondary variable is the V sample with 25% of relative error added.

$$\gamma_{primary}(h) = 0.4 + 3.7 \cdot Sph(1) \cdot \left(\frac{N157.5E}{51m}, \frac{N67.5E}{30m}\right) + 1.82 \cdot Sph(2) \cdot \left(\frac{N157.5E}{120m}, \frac{N67.5E}{35m}\right) \quad (3)$$

$$\gamma_{secondary}(h) = 2 + 3.85 \cdot Sph(1) \cdot \left(\frac{N157.5E}{51m}, \frac{N67.5E}{30m}\right) + 1.45 \cdot Sph(2) \cdot \left(\frac{N157.5E}{120m}, \frac{N67.5E}{35m}\right) \quad (4)$$

$$\gamma_{cross}(h) = 0.1 + 2 \cdot Sph(1) \cdot \left(\frac{N157.5E}{51m}, \frac{N67.5E}{30m}\right) + 1.48 \cdot Sph(2) \cdot \left(\frac{N157.5E}{120m}, \frac{N67.5E}{35m}\right) \quad (5)$$

2.2.3 Ordinary kriging

In this approach, first the two types of data were pooled together. Then, both types of information were used in the estimation using ordinary kriging. The difference in precision between the two sources of information was not considered. The variogram in this case was defined by Equation (6):

$$\gamma(h) = 1.9 + 3.6 \cdot Sph(1) \cdot \left(\frac{N157.5E}{43m}, \frac{N67.5E}{39m}\right) + 1.68 \cdot Sph(2) \cdot \left(\frac{N157.5E}{118m}, \frac{N67.5E}{42m}\right) \quad (3)$$

2.3 Comparison with the reference block grade distribution

The block estimates were compared with the reference block distribution using scatter plots and grade tonnage curves. Also, the block misclassification was assessed for each scenario. Block misclassification occurs in two situations. When the true block grade is above the cutoff grade and the estimated block grade is below, the ore block is incorrectly classified as waste. Moreover, when the true block grade is below cutoff and the estimated block grade is above, the waste block is erroneously classified as ore. Both block misclassification situations were quantified at five distinct cutoffs for each methodology.

3 RESULTS AND DISCUSSION

Figure 1 shows the scatter plots between the estimated and the true block grades. In addition, Figure 1 shows basic statistics of the two distributions and the coefficient of correlation between them.

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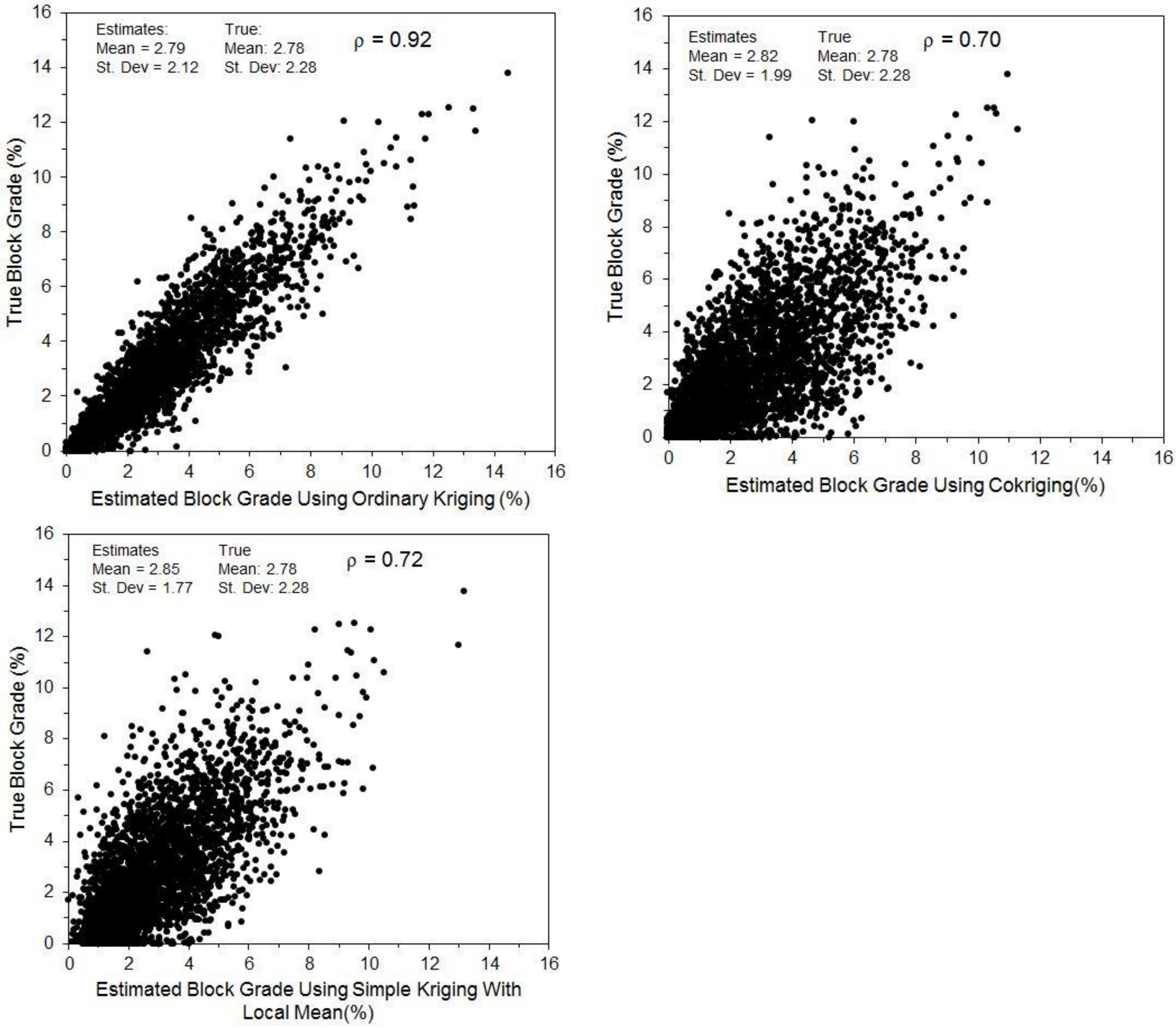

Figure 2. Scatter plot between true and estimated block grades.

Figure 1 shows that ordinary kriging generated estimated block grades much closer to the reference block grade distribution than the other two methods (cokriging and simple kriging with local varying mean). While the coefficient of correlation for ordinary kriging is 0.92, simple kriging with local mean and cokriging exhibited roughly 0.70. In addition, the standard deviation of the estimates using ordinary kriging is far more similar to the true block grades standard deviation. Simple kriging with local mean and cokriging had a higher degree of smoothing, causing a reduction in the variance of the estimates. It happened because the ordinary kriging gave more weight to the samples in the near vicinity to estimate a block. As a result, local nuances were better captured by the ordinary kriging case.

Conversely, simple kriging with local mean and cokriging gave more weight to data farther away and less weight to the samples immediately surrounding the block. This fact caused more smoothing in the estimates. The effect of smoothing resulted in a large deviation from the true block grade distribution. The highest degree of smoothing occurred in the simple kriging with local mean case. This is the method that used the least amount of information. For the ordinary kriging and the cokriging approach, sets of samples with imperfect precision were used for the block grade estimation. In contrast, the estimates with simple kriging with local mean used only one sample with poor quality. This sample was used to estimate the local mean to be

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used in Equation (1). As simple kriging with local mean used only one sample of the secondary variable, the consequence was the pronounced degree of smoothing. Another drawback of the simple kriging with local mean is that it introduced a degree of bias in the estimates. The mean of the estimates using simple kriging with local mean differs the most from the true grade. This happens because the regression line used to estimate the local mean performed poorly. Figure 1 shows that the true values depart significantly from the line used to estimate the local mean in Equation (1). The quality of the estimates using simple kriging with local mean is highly dependent on the quality of the regression line used.

The authors believe that as the samples with low precision do not contain bias (accurate but umprecise), they were accurate enough to estimate the block grade properly.

Figure 2 shows the grade tonnage curve for the reference block grade distribution and for the estimates.

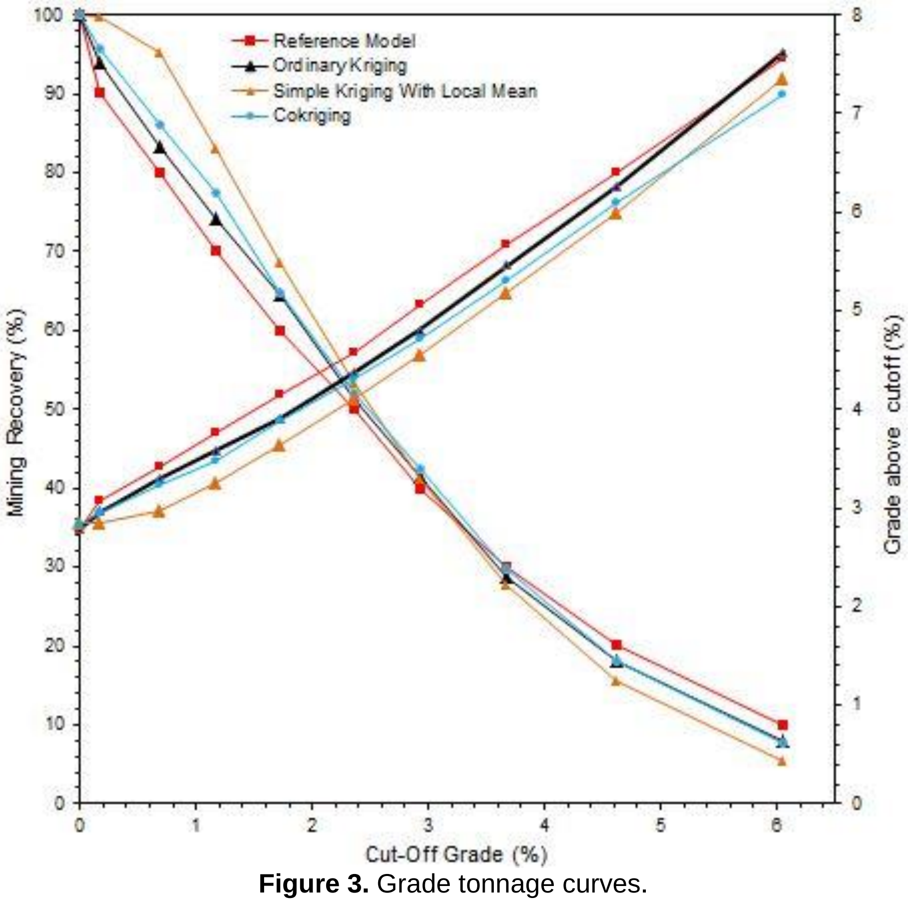

Figure 3. Grade tonnage curves.

As expected, the higher the smoothing effect, the higher the deviation from the reference grade tonnage curve. As a result, the estimates using simple kriging with local mean produced the poorer grade tonnage curve. The grade above cutoff predicted by the simple kriging with local mean approach was underestimated. Also, the largest deviations of the predicted tonnage occurred for the simple kriging with local mean. The best results were achieved with ordinary kriging. For all the cutoffs, the ordinary kriging grade tonnage curve is the closest to the reference curve.

Figure 4 shows the total number of misclassified blocks, the number of ore blocks classified as waste and the number of waste blocks classified as ore for each

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methodology. Five cutoffs were considered: 0.93%, 1.73%, 2.35%, 4.24% and 5.34%.

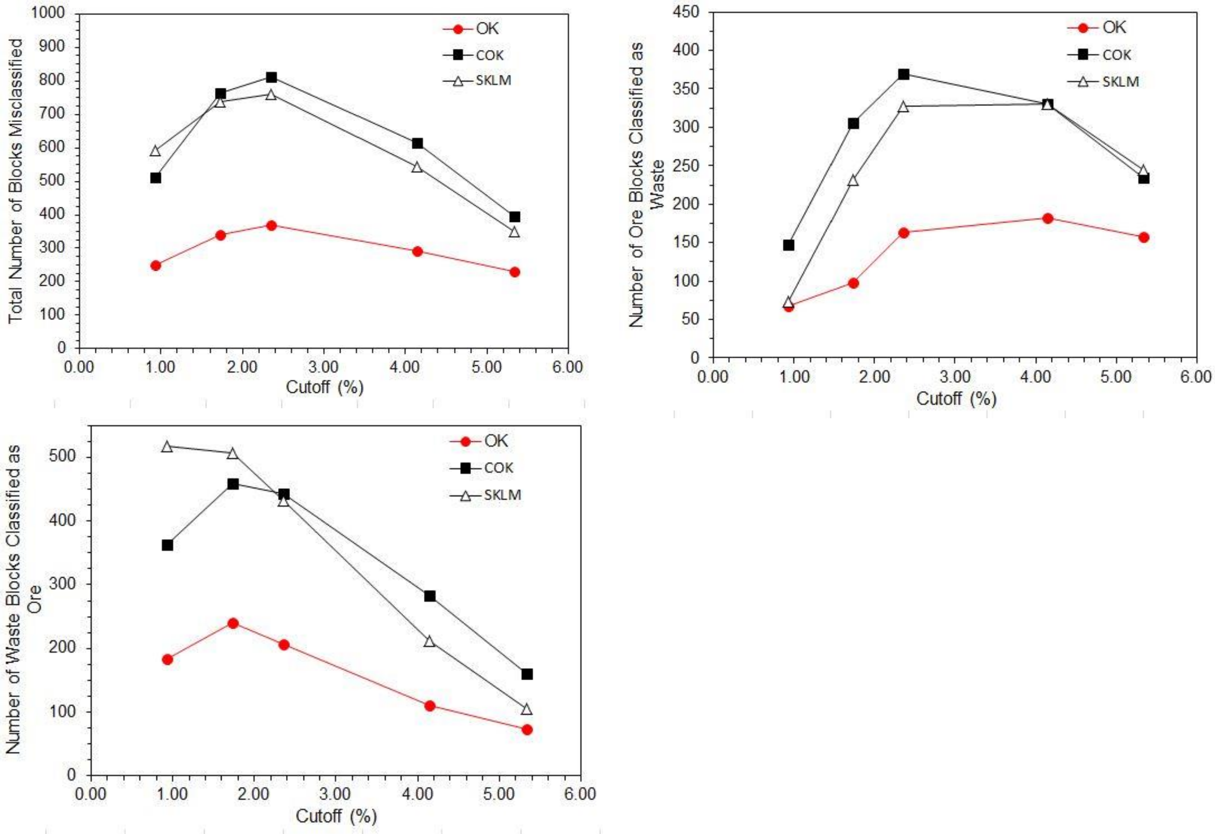

Figure 4. Block Misclassification.

Figure 4 shows that ordinary kriging generated the best result in terms of block misclassification for all the cutoffs considered. The difference is evident. At 1.73%, the number of misclassified blocks is approximately 360 using ordinary kriging. In the case of simple kriging with local mean and cokriging, the number increases to roughly 750. The authors highlight that for the cutoff of 1.73%, the tonnage of ore predicted by both ordinary kriging and cokriging were very similar (consider the grade tonnage line of ordinary kriging and cokriging, Figure 3). From a mining perspective, it means that both estimates sent approximately the same tonnage of material to the processing plant. However, the cokriging approach sent erroneously much more waste to the plant, causing dilution. Even worse, the cokriging method sent far more ore blocks to the waste pile. Similar to cokriging, the simple kriging with local mean produced poor results. The main difference is that the grade tonnage curve of the cokriging approach is more consistent with the curve of the ordinary kriging method. The better results in block misclassification shown by ordinary kriging is consistent with the scatter plots between true block values and estimated block values (Figure 2). For the ordinary kriging case, the points are less scattered.

4 CONCLUSION

Three methodologies were used to incorporate data of different precision: simple kriging with local varying mean, cokriging and ordinary kriging. The ordinary kriging approach had the least degree of smoothing, which led to estimates closer to the true block grades. The ordinary kriging resulted in a grade tonnage curve more similar to

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the reference grade tonnage curve. Also, this methodology reduced drastically the number of blocks misclassified.
As the samples with less precision do not have bias, they contributed to improve the block grade estimates. In addition, the large quantity of samples with low precision compensated their low quality. The impact of bias in the dataset on estimates should be investigated.
The use of information with low precision was compensated for reduction in the degree of smoothing. The higher the smoothing, the more severe was the block misclassification.

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