MODELING AND SIMULATION OF SENDZIMIR MILL SHAPE CONTROL ACTUATION SENSITIVITIES AND CAPABILITIES ENVELOPE WITH APPLICATIONS TO MULTIVARIABLE SHAPE CONTROL[1]

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Abstract

Intuitive decoupled notions of Sendzimir mill shape control actuator behavior do not properly depict the highly coupled, non-linear response characteristics. If applied to closed-loop shape controls, these improper notions lead to poor shape control performance and rapid infringement on actuator control limits. Accurate, fully coupled, non-linear internal models of the true actuator spatial sensitivity functions must be applied to multivariable shape controls for stable behavior to be realized. The mill's shape control actuation capabilities envelope must be considered when assessing the mill's ability to accommodate various classes of inbound shape distortions and stress distribution targets. This paper examines methods used to obtain accurate representations of the actual sensitivity functions and their mapping into the spatial context of the multivariable controls. First principle models and simulation studies are correlated with on-line shape control reactions to develop fully coupled control models that are directly applicable to singular value decomposition techniques. Real-world field studies of these modeling efforts are presented. The resulting models are embedded in singular value decomposition frameworks to create usable closed-loop controls.

Key words: Multivariable shape control

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1 INTRODUCTION

Sendzimir mills (20-high cluster arrangements) have complex, highly coupled, non-linear strip shape[1] actuation characteristics. A common actuation configuration is shown in Figure 1 and includes As-U-Roll (AUR) top crown eccentrics (B&C) and the tapered lateral 1st intermediate rolls.

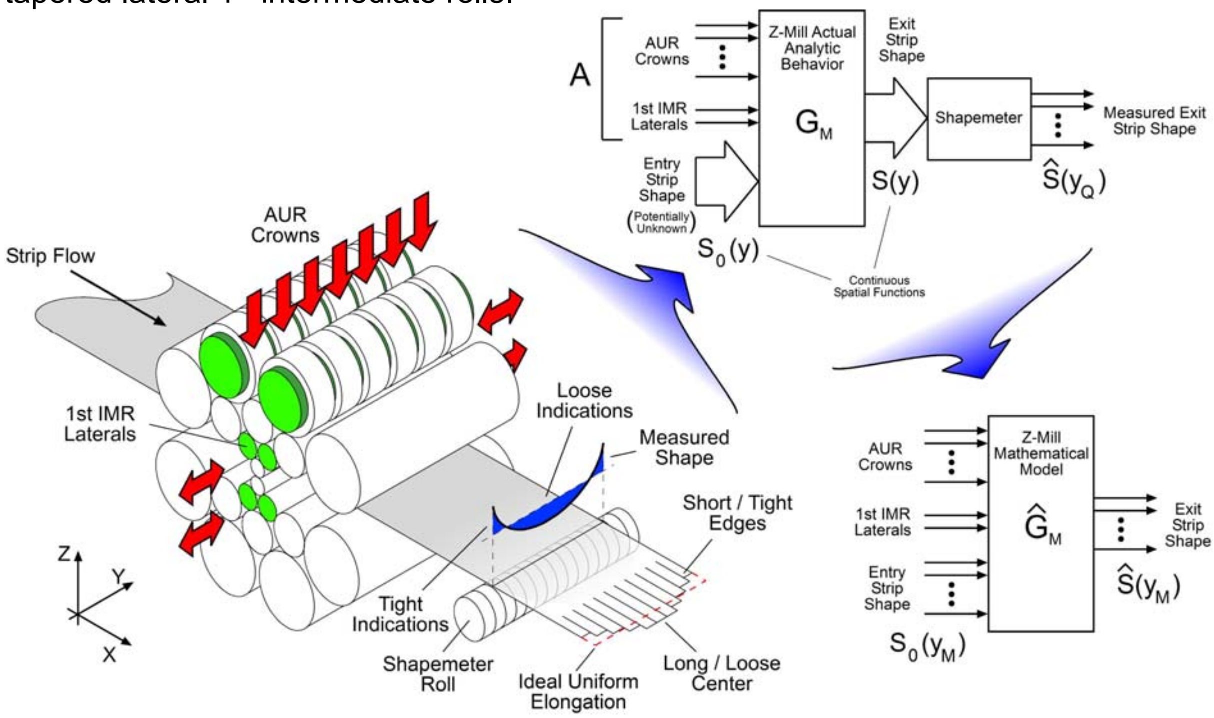

Figure 1. Illustration of the Sendzimir mill and its shape control actuators, along with the general form and structure of its characterization and modeling.

The Sendzimir mill adjusts the strip's shape by coordinating this set of actuators to provide localized corrective changes in the transverse pressure distribution (across the roll gap) that modify the localized strip elongations, thereby altering the stress patterns of the rolled strip. In changing the transverse pressure distribution, the roll cluster mechanically reacts and deforms, influencing other regions of the roll bite.
Each actuator induces a unique stress adjustment pattern on the strip's transverse stress distribution that can be characterized as a continuous spatial sensitivity function. From the ensuing roll cluster deformations, the geometry of the pattern is not localized to the vicinity of the actuator, but spans the strip width. This creates a highly coupled and potentially compromising interaction with the activities of the other actuators. The extent of the spatial frequencies of the sensitivity function is limited by the mechanical interactions within the roll cluster (e.g., roll bending, flattening, multiple contact points, etc.). To further complicate matters, these patterns change with strip width, yield stress, tension, incoming thickness, etc.

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[1] The terms "shape" and "flatness" are often used in an arbitrary or interchangeable manner, and there are no universally accepted definitions. For the purposes of this discussion, the following terms will adhere:

Shape – The transverse distribution of differential elongation induced stress within the material with respect to the material's average / nominal applied stress. This terminology implies a tensioned condition and is inherently bipolar, accounting for regions looser / longer and tighter / shorter than the nominal strip condition.

Flatness – The geometric departure of the strip from a reference plane. These distortions are associated with internal differential elongation based stress patterns that exceed the material's buckling threshold, and obtain a lower potential stress equilibrium by manifesting out of the reference plane.

Multivariable control techniques are uniquely suited for accommodating this form of non-linear, highly coupled situation.[1-5] The variability of the actuators' spatial sensitivity functions requires the controller to be either very dexterous or exceptionally robust. Robust methods[4,5] based on internal model principles encompass both requirements, in that, they are inherently robust while also employing an internal model (based on the spatial sensitivity functions) that can be adjusted to the situation at hand.

The key is to obtain an accurate model that spans not only the actuators' spatial sensitivity characteristics, but also properly describes variations in strip width, yield stress, tension, incoming thickness, etc. Beyond this, the model must be amenable to the form and format required by the chosen multivariable control technique. Figure 1 provides some insight into the nature of the mill's characterizing descriptions stemming from the true continuous spatial functions of strip stress and actuator sensitivity, to the discrete sampling grid of the available measurements.

An important factor in model development is realizing that the spatial frequency content of each actuator's sensitivity function is dominated by lower order components. The mechanical interactions within the roll cluster do not transmit localized high frequency spatial content to the roll bite. This allows each actuator to be modeled by either piece-wise continuous vectoral descriptions or a collection of low order polynomials.

The remainder of this paper discusses the combination of modeling, simulation and system identification methods used to obtain qualified models of shape control actuation behavior that are properly organized and suited for multivariable control techniques. The mill's SCCE is determined using a constrained Monte-Carlo simulation method.

2 CHARACTERIZING SENDZIMIR SHAPE CONTROL ACTUATION BEHAVIOR

As noted previously, each Sendzimir mill shape control actuator imparts a unique transverse stress adjustment distribution that can be characterized by a continuous spatial sensitivity function. Figure 2 illustrates the spatial sensitivity functions of the first four (4) AUR crown eccentric actuators of a seven (7) crown ZR23-26CN on 600mm wide strip[2]. It should be clear from Figure 2 that the actuated roll cluster deformation induced changes in the strip shape are not localized to the vicinity of the actuator, but span the strip's width. This creates a highly coupled and potentially compromising interaction with the activities of the other actuators.

[2] The consideration of only the first four (4) AUR crown eccentrics is based on an assumption of symmetry about C_4, where C_1, C_2, C_3 share folded symmetry with C_7, C_6, C_5, respectively.

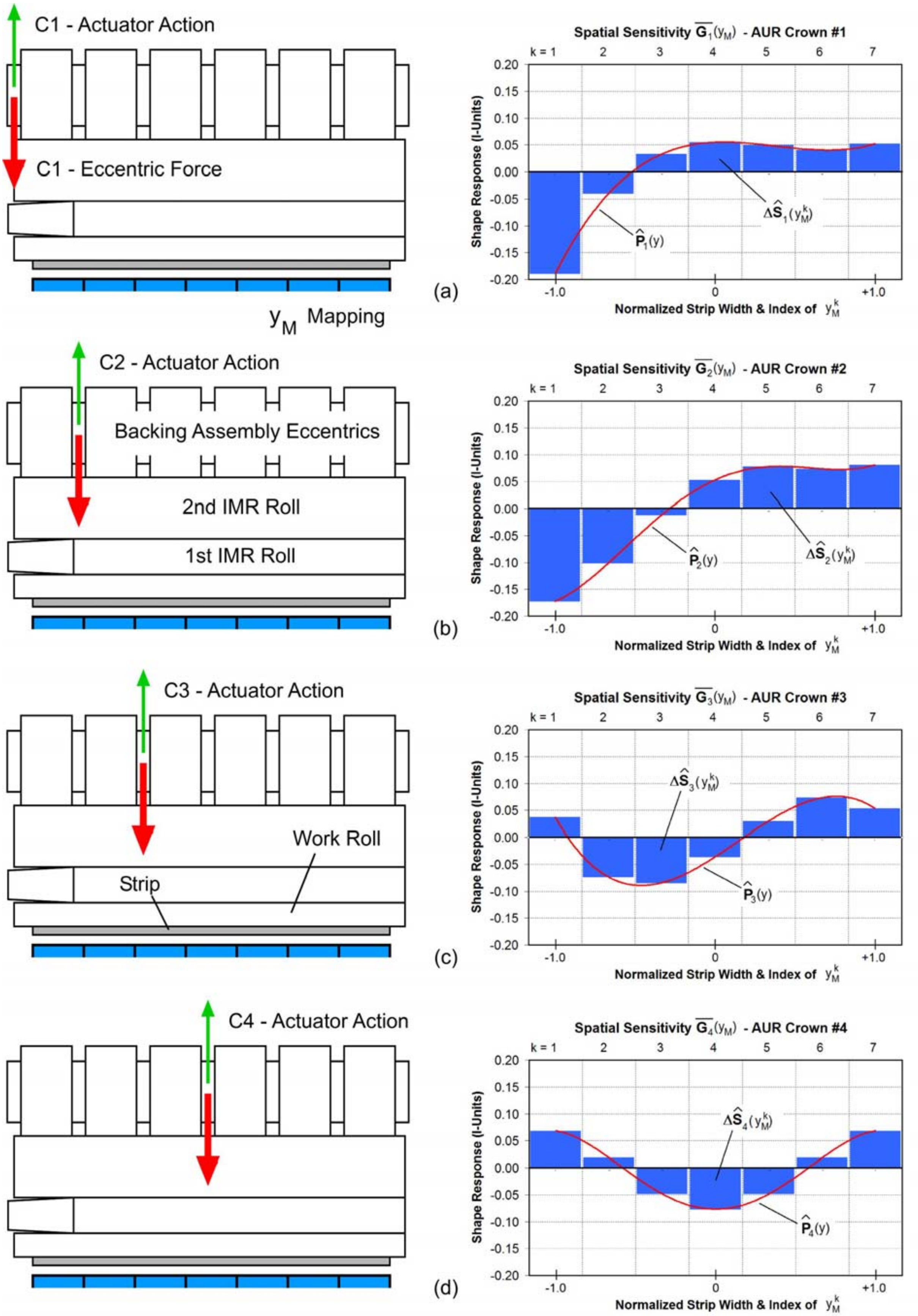


Figure 2. Illustration of the seven (7) crown ZR23-26CN Sendzimir mill's vertical roll cluster, crown actuation and resulting shape change responses for: a) Crown 1, b) Crown 2, c) Crown 3 and d) Crown 4. Analytic details of these figure components are provided in the following sections.

3 MULTIVARIABLE CONTROL MODEL FORM AND FORMAT

The foundations of many multivariable shape control methods, applicable to Sendzimir mill configurations,[1-3] are based on decompositions of the potentially complex transverse strip stress and actuation patterns (described by an internal model derived from the spatial sensitivity functions) into fundamental polynomic components (e.g., 1st, 2nd,… order curvatures). The components are typically derived from orthogonal polynomials that lead to defining a polynomial basis from which operations are carried out on these fundamental characteristics (abstracted from the mill and strip). Strip shape and actuation sensitivities are therefore decomposed into vectors of weighted polynomial order contributions. The determination of the appropriate control actions are carried out in this lower order fundamental curvature space, then transformed into the mill actuation basis for direct application to the mill. The key is to organize the internal model and its development to not only fit the accepted control format, but to also provide a direct means to vary the model to accommodate changes in strip width, yield stress, tension, incoming thickness, etc.

3.1 Actuated Shape Representation

The exit strip shape (defined previously) is represented by the continuous analytic function, $S(y)$, across the strip width, W, along the mill's transverse axis, $y \in [-W/2, +W/2]$. The exit strip shape is composed of the entry strip shape, $S_0(y)$, and the changes imparted by the mill's shape adjusting actuation, $\Delta S(y)$.

$$S(y) = S_0(y) + \Delta S(y) \tag{1}$$

Our modeling efforts will consider piece-wise continuous approximations, $\Delta\hat{S}(y_\alpha)$, of $\Delta S(y)$, evaluated at a set of uniformly distributed discrete points that span the normalized strip width over the interval $[-1,+1]$:

$$y_M = \left\{y_M^0, y_M^1, \ldots, y_M^{N-1}\right\} \quad \text{and} \quad y_S = \left\{y_S^0, y_S^1, \ldots, y_S^{Q-1}\right\} \quad \text{with } Q > N \tag{2}$$

where N is the number of mill shape actuators and Q is the number of active shapemeter measurement zones spanning the given strip width. Uniform distribution of these sets is provided by:

$$y_\alpha^k = y_\alpha^{k-1} + \frac{2}{\beta} \quad \text{for } k = 1,2,\ldots,\beta\text{-}2 \tag{3}$$

having boundary conditions:

$$y_\alpha^0 = -1 \quad \text{and} \quad y_\alpha^{\beta-1} = +1 \tag{4}$$

where $(\alpha,\beta) = (M,N)$ or (S,Q), respectively. The resulting sampled data / piece-wise approximation of the actuated changes in strip shape is therefore given by:

$$\Delta S(y) \sim \Delta\hat{S}(y_\alpha) = \left[\Delta\hat{S}(y_\alpha^0), \Delta\hat{S}(y_\alpha^1), \ldots, \Delta\hat{S}(y_\alpha^{\beta-1})\right]^T \tag{5}$$

The spatial sensitivity functions (required in model development) are derived from a combination of both modeled and on-line shapemeter measured depictions of $\Delta\hat{S}(y_S)$ transformed by polynomial fitting to the reduced spatial frequency (and model ready) $\Delta\hat{S}(y_M)$.

3.2 Mill Actuator Representation

The mill's shape actuators are composed of N_C AUR crown eccentrics ($N_C = 7$ in the case study of Figure 2) and two (2) 1st IMR tapered lateral rolls, whose operating states are represented by the $N = N_C + 2$ elements of the vector, $A \in \mathbb{R}^N$:

$$A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N_C} \\ a_{N-1} \end{bmatrix} = \begin{bmatrix} L_T \\ C_1 \\ C_2 \\ \vdots \\ C_{N_C} \\ L_B \end{bmatrix} \cong \begin{bmatrix} \text{Top / Front 1}^{st}\text{ IMR} \\ \text{AUR Crown \#1} \\ \text{AUR Crown \#2} \\ \vdots \\ \text{AUR Crown \#}N_C \\ \text{Bottom / Rear 1}^{st}\text{ IMR} \end{bmatrix} \tag{6}$$

The range of the elements of A, are normalized over the interval $[-1,+1]$.

3.3 Mill Internal Model Representation

The mill's internal model (for closed-loop control applications) is represented by an NxN square static matrix, $\hat{G}_M \in \mathbb{R}^{NxN}$, that transforms the mill shape actuator activity, A, to $\Delta\hat{S}(y_M)$, the approximated mill induced exit strip shape reactions, $\Delta S(y)$, by:

$$\Delta S(y) = S(y) - S_0(y) : \Delta\hat{S}(y_M) = \hat{G}_M A$$

$$= \begin{bmatrix} \Delta\hat{S}(y_M^0) \\ \Delta\hat{S}(y_M^1) \\ \vdots \\ \Delta\hat{S}(y_M^{N-1}) \end{bmatrix} = \left[\overline{G_0}(y_M) \;\middle|\; \overline{G_1}(y_M) \;\middle|\; \cdots \;\middle|\; \overline{G_{N_C}}(y_M) \;\middle|\; \overline{G_{N-1}}(y_M) \right] \begin{bmatrix} L_T \\ C_1 \\ C_2 \\ \vdots \\ C_{N_C} \\ L_B \end{bmatrix} \tag{7}$$

where $\hat{G}_M$ is valid about a given operating point with the assumption that the strip is centered along the mill's transverse rolling axis. From Eq(7), $\hat{G}_M$ is organized as a collection of column vectors:

$$\overline{G_i}(y_M) = \left[g_i^0 \quad g_i^1 \quad \cdots \quad g_i^k \quad \cdots \quad g_i^{N-1} \right]^T \tag{8}$$

where $\overline{G_i}(y_M)$ is the approximation of the spatial sensitivity function that maps the activity of the ith actuator, a_i, to changes in the strip shape, $\Delta\hat{S}(y_M)$, over y_M, having elements:

$$g_i^k = \frac{\Delta\hat{S}(y_M^k)}{\Delta a_i} : \left.\frac{\partial S(y)}{\partial a_i}\right|_{y = \frac{W}{2} y_M^k} : \left.\frac{\Delta S(y)}{\Delta a_i}\right|_{y = \frac{W}{2} y_M^k} \tag{9}$$

To preserve the zero mean definition of shape [1] and to be non-interactive with the gauge control system, the elements of $\hat{G}_M$ must adhere to:

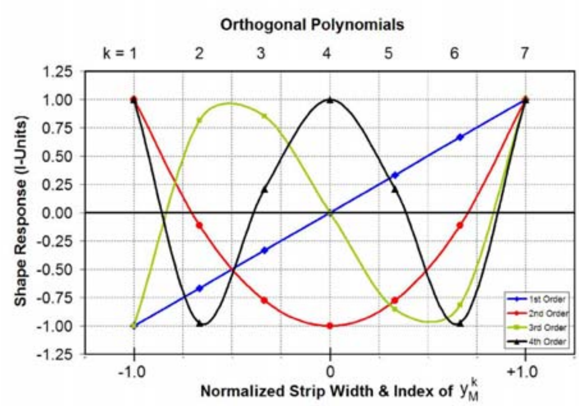
$$\sum_{i=0}^{N-1} g_i^k = 0 \;\; \forall\, k \quad \text{and} \quad \sum_{k=0}^{N-1} g_i^k = 0 \;\; \forall\, i \tag{10}$$

3.4 Closed-Loop Control Organization

It is important to coordinate the model development with the multivariable control application's intents and needs.[1-5] In the most simplistic case, the control law design involves direct model inversion, $\hat{G}_M^{-1}$. Unfortunately, $\hat{G}_M$ is not necessarily of full rank and therefore alternative methods must be employed.

The smooth spatial curvatures of the sensitivity functions allows lower order parameterization of the shape profiles by means of polynomial transformations [6], which also has the benefit of reducing the resulting controller's complexity (i.e., fewer control loops to close). The approach taken here, is to form the parameterization on a set of orthogonal polynomials, $P(y) = \{P_1(y), P_2(y), \ldots, P_V(y)\}$, that develop a set of orthonormal basis functions associated with the fundamental curvature components, $\$_\varphi^i(t)$, of the measured strip shape, the shape target and mill actuator influences, where $\varphi = S, T, M$, respectively and $i = 1,2,\ldots,V$.

From the constraints of Eq(10) we can see that 0^{th} order behavior is excluded. Based on experimental and simulation results along with certain physical considerations, up to a 4^{th} order polynomic behavior will be employed (V = 4). Using Chebyshev orthogonal polynomials, $P(y)$ is given by:



$$P_1(y) = y \qquad (11a)$$

$$P_2(y) = 2y^2 - 1 \qquad (11b)$$

$$P_3(y) = 4y^3 - 3y \qquad (11c)$$

$$P_4(y) = 8y^4 - 8y^2 + 1 \qquad (11d)$$

Figure 3. Chebyshev orthogonal polynomials used in fundamental curvature component parameterization.

As an example, consider the parameterization of the time evolution of mill induced strip shape changes, $\Delta\hat{S}(y_M^k, t)$, given by:

$$\Delta\hat{S}(y_M, t) \; : \; \sum_{i=1}^{4} \$_S^i(t)\, P_i(y_M) \qquad (12a)$$

$$\Delta\hat{S}(y_M, t) \; : \; \bar{P}(y_M)\, \$_S(t) = \begin{bmatrix} P_1(y_M^0) & P_2(y_M^0) & P_3(y_M^0) & P_4(y_M^0) \\ P_1(y_M^1) & P_2(y_M^1) & P_3(y_M^1) & P_4(y_M^1) \\ \vdots & \vdots & \vdots & \vdots \\ P_1(y_M^{N-1}) & P_2(y_M^{N-1}) & P_3(y_M^{N-1}) & P_4(y_M^{N-1}) \end{bmatrix} \begin{bmatrix} \$_S^1(t) \\ \$_S^2(t) \\ \$_S^3(t) \\ \$_S^4(t) \end{bmatrix} \qquad (12b)$$

with the extension to higher spatial frequency shapemeter measurements by $\bar{P}(y_S)$.

The terminology is reduced by:

$$\bar{P} = \bar{P}(y_M) \quad \text{and} \quad \bar{P}_0 = \bar{P}(y_S) \qquad (13)$$

It follows that $\left(\overline{P}\right)^{T}\overline{P} = \left(\overline{P}_0\right)^{T}\overline{P}_0 = I_V$ for orthogonal polynomials, therefore the polynomial decomposition of the fundamental curvature components is given by:

$$\$_S(t) = \left[\left(\overline{P}\right)^{T}\overline{P}\right]^{-1}\left(\overline{P}\right)^{T}\Delta\hat{S}(y_M,t) = \left(\overline{P}\right)^{T}\Delta\hat{S}(y_M,t) \qquad (14)$$

Using this parameterizing / decomposing transformation, it is possible to formulate the control problem in terms of the fundamental curvature components. Here, the problem is posed as one of causing the curvature components of the measured exit strip shape, $\$_S(t)$, to equal the curvatures of the shape target reference, $\$_T(t)$, in terms of the curvatures of the modeled mill, $\hat{G}_M$, $\hat{G}_\$$, which corresponds to the curvatures of the shape error, $\$_E(t)$, being driven to zero. The controller arrangement is given by:

$$\$_S(t) = \begin{bmatrix} \$_S^1(t) \\ \vdots \\ \$_S^4(t) \end{bmatrix} = \left(\overline{P}_0\right)^{T} S_S(y_S,t) \quad \rightarrow \quad S_S(y_S,t) \;@\; \begin{matrix}\text{Shapemeter}\\ \text{Measurements}\end{matrix} \qquad (15a)$$

$$\$_T(t) = \left(\overline{P}_0\right)^{T} S_T(y_S,t) \qquad (15b)$$

$$\$_E(t) = \$_T(t) - \$_S(t) \qquad (15c)$$

$$\hat{G}_\$ = \left(\overline{P}\right)^{T}\hat{G}_M\,\overline{P} \qquad (15d)$$

where $\hat{G}_\$ \in \mathbb{R}^{VxV}$ is a full rank VxV matrix that transforms the curvature components of the mill actuators spatial sensitivity functions, $\$_A(t)$, to the curvatures of the induced strip shape changes $\$_{\Delta S}(t)$. $\hat{G}_\$$ is basically a spatial filter that reduces complexities of the actual mill to its most fundamental curvature components. The full rank of $\hat{G}_\$$ allows the controller to be based on $\left(\hat{G}_\$\right)^{-1}$ and thereby overcomes the potential rank limitations of $\hat{G}_M$. The controller shown in Figure 4 and is given by:

$$G_C = \overline{P}\left(\hat{G}_\$\right)^{-1} \qquad (16)$$

$$A(t) = \overline{P}\,\$_A(t) = G_C\$_E(t) \quad \rightarrow \quad \$_A(t) = \left(\hat{G}_\$\right)^{-1}\$_E(t) \qquad (17)$$

The most important outcome of this controller organization is that the required modeling of the mill actuator's spatial sensitivity functions (for developing $\hat{G}_M$) is directly tied to the order of curvature employed in $\overline{P}$ and $\overline{P}_0$ (i.e., V = 4). This will guide the manner in which $\hat{G}_M$ derived.

3.5 Model Adaption

It is necessary for the internal model to accommodate changes in strip width, yield stress, tension, incoming thickness, etc., which can have a dramatic effect on the nature of the spatial sensitivity functions, $\overline{G_i}(y_M)$. Figure 5 shows how these changes can impact the spatial responses.

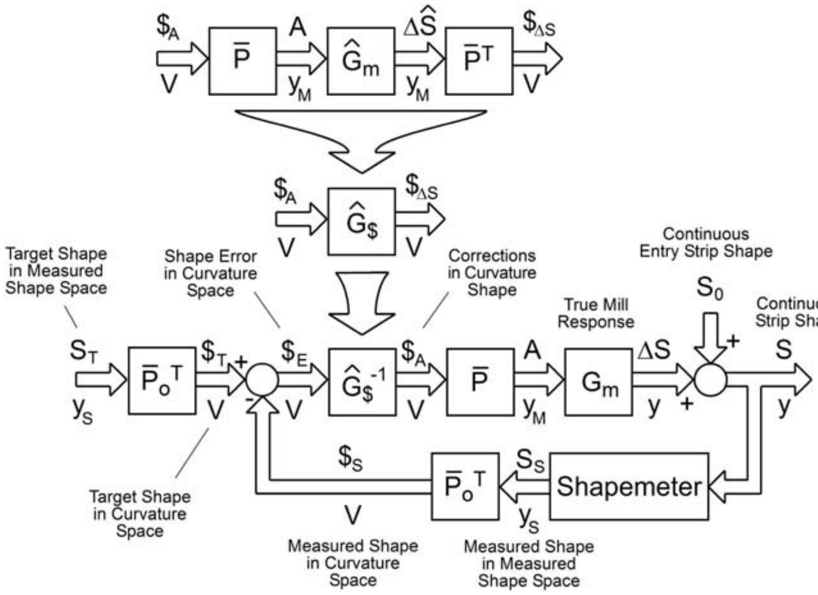


Figure 4. Block diagram illustration of the overall closed-loop control system configuration.

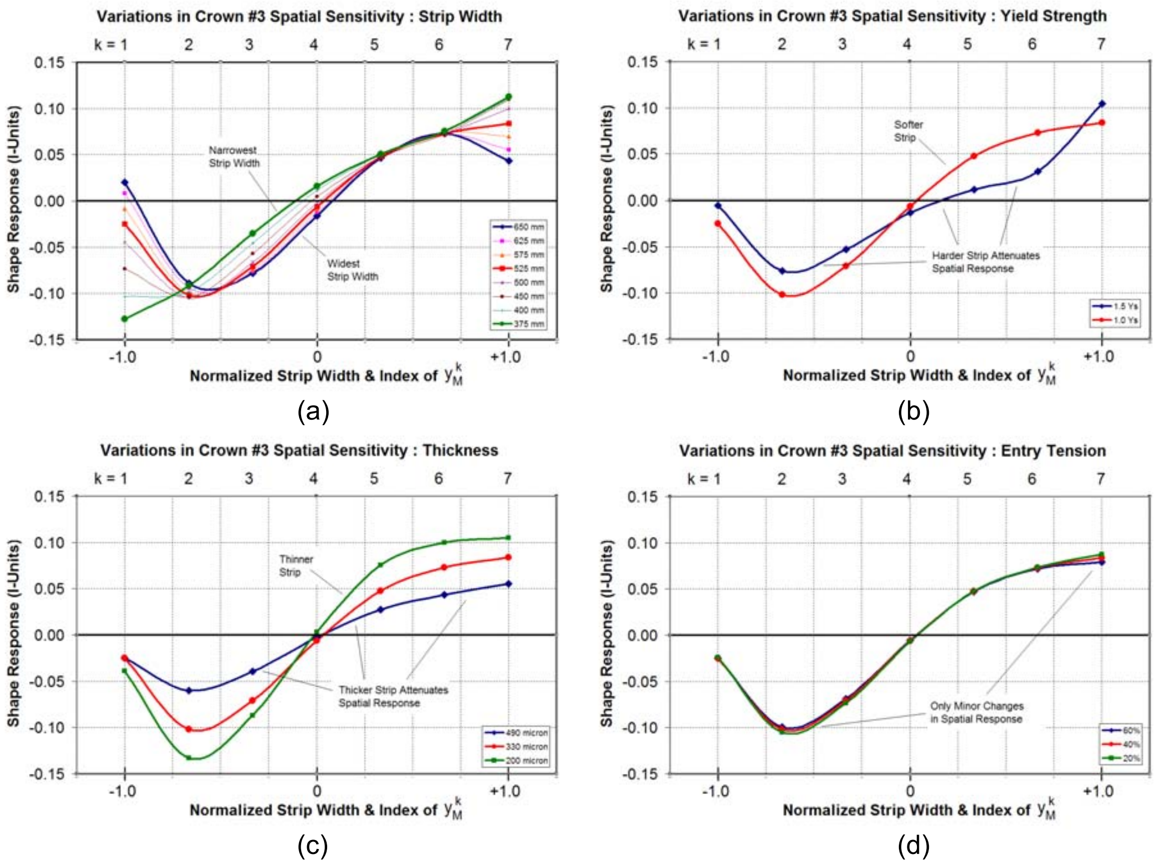


Figure 5. Illustration of the variations in Crown #3's spatial sensitivity as functions of : a) Strip Width, b) Yield Strength, c) Exit Thickness, d) Entry Tension.

To accommodate changes in the spatial sensitivity functions, $\overline{G_i}(y_M)$, the elements of Eq(8, 9) are extended by the multi-dimensional surface functions:

$$g_i^k = \frac{\Delta\hat{S}\left(y_M^k, \Delta W, \Delta Y_S, \Delta G_X, \Delta T_E\right)}{\Delta a_i} \tag{18}$$

where $\Delta\hat{S}\left(y_M^k,\Delta W,\Delta Y_S,\Delta G_X,\Delta T_E\right)$ is a multivariable polynomial surface mapping of the variations at each, y_M^k (derived from the variations shown in Figure 5). At a minimum, the g_i^k's of all $\overline{G_i}\left(y_M\right)$'s in $\hat{G}_M$ are computed prior to the initiation of a given pass, however, this can be applied to dynamically adaptive arrangements.

4 RESPONSE DECOMPOSITION AND MODELING DEVELOPMENT

There are many ways to formulate the internal model, $\hat{G}_M$, including the use of first-principals advanced analytic simulation models of Sendzimir mill behavior,[7,8] system identification techniques, and roll bite deformation tests.[9] The approach taken has been to first employ advanced analytic simulations models[7,8] to define the "expected" spatial sensitivity characteristics, $\overline{G_i^E}\left(y_S\right)$. These simulated characteristics are then refined by comparison with the results of on-line (direct shapemeter based) system identification techniques (using probative a_i's) that describe the particular mill's actual spatial sensitivity function, $\overline{G_i^S}\left(y_S\right)$. The result of this refinement is a piecewise continuous representation, $\overline{G_i}\left(y_S\right)$, that is of higher spatial frequency than $\overline{G_i}\left(y_M\right)$ needed to construct the model, $\hat{G}_M$. A power series polynomial, $\hat{P}_i\left(y\right)$ (see Figure 2), is the Least Squares Fit (LSF) of $\overline{G_i}\left(y_S\right)$, from which $\overline{G_i}\left(y_M\right)$ is obtained by evaluation:

$$\overline{G_i}\left(y_M\right) = \hat{P}_i\left(y\right)\Big|_{y=y_M} \;\Rightarrow\; g_i^k = \hat{P}_i\left(y_M^k\right) \tag{19}$$

where the order of the $\hat{P}_i\left(y\right)$'s need not exceed that of $\overline{P}$, $\overline{P}_0$ (i.e., V = 4). Figure 6 illustrates this process of model development.

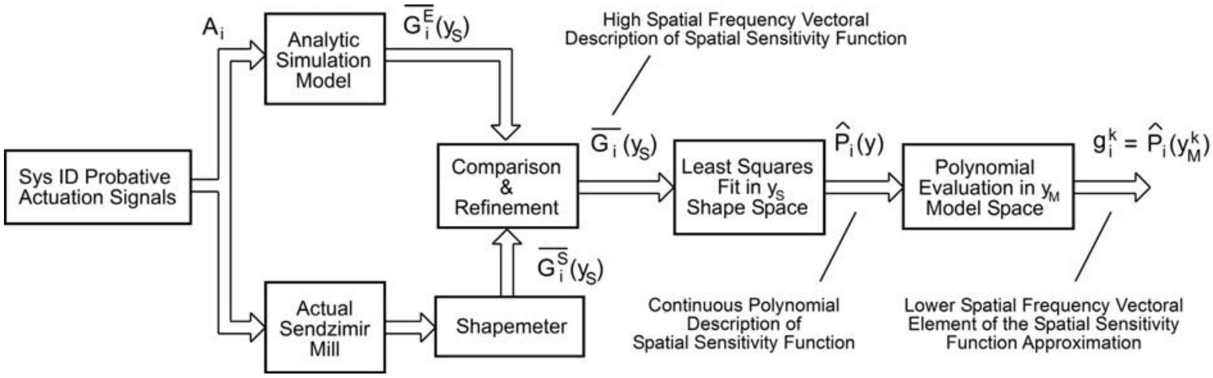


Figure 6. Block diagram of the internal model development process.

5 CONCLUSION AND COMMENTARY

A method used to obtain accurate representations of the actual sensitivity functions and their mapping into the spatial context of the multivariable controls has been presented. First principle models and simulation studies are correlated with on-line shape control reactions to develop fully coupled control models having the transverse spatial resolution of the shapemeter. High spatial frequency vectoral descriptions of the actual actuator's spatial sensitivity functions are directly obtained the model results. Least Squares Fitting of the vectors produce higher order polynomials describing the sensitivity functions independent of spatial frequency. Evaluation of these polynomials at the spatial grid of the actuators provides vectoral elements that map the actuation space to shape adjustment space. Combining these elements along vectoral lines establishes a matrix representation of this spatial transformation, and is directly applicable to multivariable control design and implementation techniques.

The focus of current developmental work involves using these techniques in combination with Monte Carlo simulation methods, to develop descriptions of the mill's Shape Correction Capabilities Envelope (SCCE). The SCCE defines the extent of shape correction that the mill can provide at a given situation and instance. The SCCE allows one to determine (in advance) whether the current mill set-up can attain a selected target in the presence of an understood incoming material stress distribution. On-line SCCE predictions for target selection and roll cluster set-up are currently under test, and providing interesting insight into the mills behavior patterns and avenues of performance improvement and optimization.

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