ISSN 2179-3956

THE EFFECT OF PERMEABILITY DISTRIBUTION IN THE NUMERICAL ANALYSIS OF AEROSTATIC CERAMIC POROUS BEARINGS[1]

Rodrigo Nicoletti[2]
Benedito de Moraes Purquerio[3]
Zilda de Castro Silveira[2]

Abstract

This work presents a numerically study about effect of permeability in the static characteristics of aerostatic ceramic porous bearings. This study is important for bearing design in function of inherent alterations during manufacturing and sintering processes that can induce variability in the permeability among porous ceramic samples. The mathematical modeling of such bearings is based on the modified Reynolds equation, which depends on the pressure-flow assumption (Darcy or Forchheimer) on the static and dynamic permeability coefficients of the porous matrix. In this work, the modified Reynolds equation is numerically solved considering the variations of permeability values, whose distribution was obtained from experimental tests of the ceramic samples. Monte Carlo method is used to introduce variability in the solutions. One focuses on the effects of such variation in the results of bearing loading capacity and bearing stiffness coefficient. Variations in dynamic permeability affect system linearity, which is quite obvious considering that this parameter only appears when quadratic pressure-flow assumption is adopted (system is non linear). However, variations in permeability which is adopted when linear pressure-flow assumption is considered, may lead to a system non-linearity if design variables will be in the edge of the linear region. Therefore, optimum regions of bearing load capacity and stiffness may vary significantly due to static permeability variation, thus hindering bearing design.

Keywords: Ceramic porous; Aerostatic bearings; Monte Carlo simulation; Permeability.

O EFEITO DA DISTRIBUIÇÃO DA PERMEABILIDADE NA ANÁLISE NUMÉRICA DE MANCAIS AEROSTÁTICOS CERÂMICOS POROSOS

Resumo

Este trabalho apresenta o estudo numérico do efeito da variação da permeabilidade nas características estáticas de mancais cerâmicos porosos. Essa investigação é importante para o projeto de mancais, devido às eventuais alterações na variação da permeabilidade entre as amostras do material cerâmico, mesmo quando há repetibilidade do processo, durante a manufatura da matéria-prima, e posterior processo de sinterização, O modelo matemático dos mancais está baseado na equação modificada de Reynolds, que depende da hipótese de escoamento-pressão adotada (Darcy ou Forchheimer) sobre o coeficiente de permeabilidade estático e dinâmico na matriz porosa. Neste trabalho a equação modificada de Reynolds é solucionada numericamente considerando a variação dos valores de permeabilidade, cujas distribuições foram obtidas a partir de ensaios experimentais com as amostras do material cerâmico poroso. O método de Monte Carlo é utilizado para introduzir variabilidade nas soluções, cujo objetivo é obter os efeitos de tais variações, nos resultados da capacidade de carga do mancal e de seu coeficiente de rigidez. Variações na permeabilidade dinâmica afeta linearmente o sistema, condição aceitável uma vez que esse parâmetro somente tem influência no sistema, quando o termo quadrático do escoamento-pressão é considerado no sistema. Entretanto, variações na permeabilidade estática (adotada), quando o termo escoamento-pressão é linear pode conduzir o sistema, a um comportamento não linear, se as variáveis de projeto estiverem no contorno da região linear. Dessa forma, regiões ótimas de capacidade de carga e rigidez do mancal podem variar significativamente, devido às alterações na permeabilidade estática inviabilizando o projeto do mancal.

Palavras-chave: Cerâmica porosa; Mancal aerostático; Simulação de Monte Carlo; Permeabilidade.

[1] *Technical contribution to the First International Brazilian Conference on Tribology – TriboBr-2010, November, 24th-26th, 2010, Rio de Janeiro, RJ, Brazil.*

[2] *Assistant Professor. Engineering School of Sao Carlos, Departament of Mechanical Engineering, University of Sao Paulo.*

[3] *Professor. Engineering School of Sao Carlos, Departament of Mechanical Engineering, University of Sao Paulo.*

ISSN 2179-3956

1 INTRODUCTION

Aerostatic bearings applications increased significantly in the last years particularly due to high and ultra precision devices and machines usually indicated in slide ways and spindles used to manufacturing of microelectronic and Micro-Electro-Mechanical Systems (MEMS), with large application in the nanotechnology field. The performance of an ultra precision machine directly depends on the static and dynamic behavior of its components, such as precision, stiffness, movement repeatability and load capacity. The rotor-bearing pair in such machines plays an important role in setting the fundamental characteristics just mentioned. In this case, aerostatic bearings are suitable for spindle and linear guide applications due to their inherent characteristics. The use of air as the lubrication fluid in the bearings brings several operational advantages such as the low wear and noise between moving parts, low contamination, high movement precision and repeatability, high load capacity, feasibility of achieving high rotating speeds, and long component life. The use of porous ceramic materials as an alternative design to the orifices and pockets restrictors of the conventional metallic aerostatic bearing, presents a simple design and manufacturing solution, with the further advantage of supporting higher temperatures without significant dimensional variation.
The design of machine rotating depends of operational range of the rotor-bearing system that involves estimation of the loading capacity, friction coefficient, power loss, stiffness and damping equivalent coefficients).(1) These characteristics can be estimated from the numerical solution of the modified Reynolds equation (1), which represents the behavior of the fluid in the rotor-bearing interface (aerostatic porous bearing). This equation considers geometric parameters (bearing clearance, rotor radius, bearing length and radius), fluid parameters (dynamic viscosity, density, temperature), operational parameters (rotor speed), and porous material parameters (static and dynamic permeabilities). The adoption of nominal values for these parameters of the modified Reynolds equation is very common in the design of such bearings. However, this practice results in a single solution of the equation (pressure distribution of the lubricating fluid), thus resulting in single values of the rotor-bearing characteristics, which will define the operational range of the system. On one hand, one must be very confident in the adopted parameter values to trust results. On the other hand, the adoption of nominal values will not allow sensitivity as well as value uncertainty analyzes.
In literature, one can find different sensitivity analyzes of the rotor-bearing system to parameter variations. Hannon, Braun and Hariharan(2) adopt a modified Reynolds equation that considers bearing surface curvature, but mainly variability of parameters such as density, viscosity, and specific heat of the fluid as a function of temperature. Chang and Zhao(3) present a sensitivity analysis in mixed lubricated bearings (oil plus contact), where the Reynolds equation is solved for different parameter values together with the contact model. Similar approach is presented by Chun(4) in a thermo hydrodynamic analysis of the bearing. A broad study of tilting-pad bearing sensitivity is presented by Fillon, Dmochowski and Dadouche(5) where variations of pre-load factor, assembling clearance and pivot positions are investigated. The influence of manufacturing dimensional variation of bearings is investigated in Ogrodnik et al.(6) where the authors focus on dynamic stability of the rotor-bearing system.
The Monte Carlo method is an established method for investigating system sensitivity to parameter variation, requiring the exhaustive solution of the equations that

ISSN 2179-3956

represent the system.[7].An application of this method in the sensitivity analysis of bearings can be found in Wang et al.,[8] where the steel case of a four cylinder engine, the crank shaft and the four hydrodynamic bearings are modeled by metamodels, and sensitivity to manufacturing dimensional variation is investigated.

The Reynolds equation that represents the air flow in aerostatic porous bearings depends on the injection velocity of air in the bearing clearance. This injection velocity depends of the adopted hypothesis for the flow (pressure-flow assumption) in the porous matrix that composes the bearing casing, which can be linear or quadratic. In both cases, permeability is a parameter that must be known and of fundamental importance in defining rotor-bearing characteristics. In literature, a few studies on the variability of material permeability of porous bearings can be found, focusing on material anisotropy and air flow variations within the porous matrix.[9-11] Other works focus on detailed material modeling, where distribution, diameter and connectivity of pores are considered in a probabilistic analysis Mann, Androutsopoulos and Golshan[12] and Petropoulos, Petrou and Kanellopoulos.[13] D'Agostino, Ruggiero and Senatore[14] investigate rotor-bearing stability as a function of material permeability, however not in a probabilistic way.

Numerical results are presented based on the model considering material permeability distribution, obtained from experimental tests of ceramic samples. Monte Carlo method is used to introduce variability in the solutions. The purpose of this work is research the effects of such variability on the predictions of bearing loading capacity and bearing equivalent stiffness coefficients.

2 MATHEMATICAL MODELING

Aerostatic porous bearings are air lubricated bearings, where the supporting mechanism of the shaft is pressurized air. Pressurized air is injected through the porous matrix (bearing casing) and defines the fluid pressure distribution in the rotor-bearing interface as Figure 1.

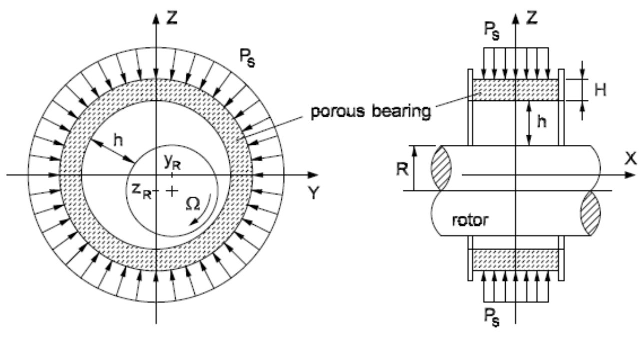


Figure 1 – Schematic view of the aerostatic porous bearing.[1]

The modified Reynolds equation that represents the fluid behavior in the rotor-bearing interface of an aerostatic porous bearing is given by:

$$\left(\frac{2L_t}{L}\right)^2 \frac{\partial}{\partial \bar{x}}\left(\bar{p}\bar{h}_3 \frac{\partial \bar{p}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{p}\bar{h}_3 \frac{\partial \bar{p}}{\partial \bar{y}}\right) = \Lambda \frac{\partial\left(\bar{p}\bar{h}\right)}{\partial \bar{y}} + \Psi \frac{\partial\left(\bar{p}\bar{h}\right)}{\partial \tau} + \Gamma \frac{\bar{p}}{\bar{p}+1}\left(\Phi - \sqrt{\Phi^2 + 2\Phi(1-\bar{p}^2)}\right) \quad (1)$$

ISSN 2179-3956

where L and L_t are the bearing width and inner perimeter, respectively; $\bar{p}$ is the non dimensional pressure (p/P_s); h is the non dimensional clearance *(h/c)*; *c* is the assembled clearance; P_s is the air supply pressure; τ is the non dimensional time *(ω.t)*; $\bar{x}$ and $\bar{y}$ are non dimensional coordinates in axial *(2x/L)* and tangential (y/L_t) directions, respectively. The non dimensional parameter Λ is related to the rotating velocity of the shaft, as follows:

$$\Lambda = \frac{6\eta U L_t}{c^2 P_s} \tag{2}$$

Where η, is the air dynamic viscosity and *U* is the velocity of the shaft surface. The non dimensional parameter ψ is related to shaft radial dynamic excitation, as follows:

$$\Psi = \frac{12\eta\omega L_t^{\ 2}}{c^2 P_s} \tag{3}$$

Where ω is the excitation frequency of the shaft. The non dimensional parameter Γ is related to the porous material that composes the bearing casing, as follows:

$$\Gamma = \frac{12 k_1 L_t^{\ 2}}{c^3 r_i \ln(r_o / r_i)} \tag{4}$$

Where k_1 is the viscous permeability coefficient of the porous material, and r_i and r_0 are the inner and outer radius of the porous bearing casing, respectively. The non dimensional parameter Φ is related to the linearity of the pressure-flow relationship in the porous material, as follows:

$$\Phi = \frac{k_2 \Re T \ln^2(r_o / r_i)\eta^2}{k_1^{\ 2} P_s^{\ 2}(1/r_i - 1/r_o)} \tag{5}$$

Where k_2 is the inertia permeability coefficient of the porous material; $\Re$ is the gas constant, and *T* is the air temperature in the bearing gap (considered constant in this work).

As one can see the parameters Γ and Φ are related to the porous material of the bearing and, consequently, are strongly affected by material permeability. There are two permeability coefficients to be considered: the viscous and the inertia coefficients. The viscous permeability coefficient (k_1) is obtained from a linear assumption of the pressure-flow relationship of the fluid crossing the porous material. The inertia permeability coefficient (k_2) is obtained from a quadratic assumption of the pressure-flow relationship of the fluid crossing the porous material. If the system tends to linearity, parameter Φ tends to infinity. Results presented in Nicoletti, Silveira and Purquerio [(1)] shows that system can be considered linear for Φ >10.

ISSN 2179-3956

3 MATERIAL MANUFACTURING AND PERMEABILITY DISTRIBUTION

A set of samples was obtained using different concentrations of pore-forming agent and different time of milling, in order to obtain a porous ceramic structure. The materials used to obtain the porous ceramic matrices were: alumina A 1000 SG (Almatis) with diameter of 0.5 um; sucrose (as pore-forming agent); Polivinil-Butirol (PVB) for superficial adhesion, resin and isopropyl alcohol. The barbotine was composed by 30% in volume by alumina plus sucrose, and 70% with isopropyl alcohol and PVB. The mixture and homogenization of the barbotine used to obtain the ceramic powder were made with a mill of balls. The circular material samples were conformed to final dimensions of the 32 mm and thickness of 5 mm as showed in Figure 2 and 3. The conformation processes to obtain bearing samples are the uniaxial and cold isostatic pressing.

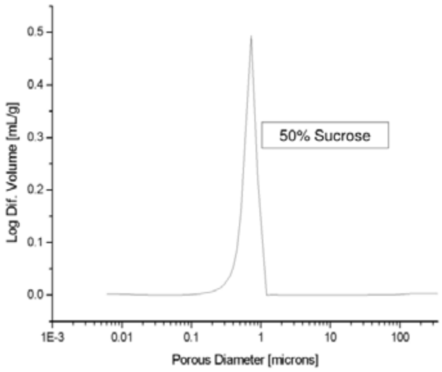
Figure 2 - Mould and sample bearing conformed uniaxilly.

Figure 3 - Mould and sample bearing conformed uniaxilly.

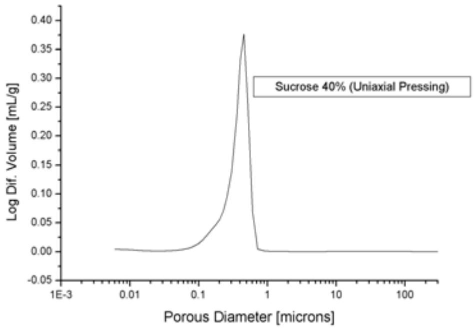
The determination of size and distribution of the pores were analyzed by Scanning Electron Microscope (SEM) and Mercury Porosimeter. For Mercury Porosimeter analysis to the sucrose concentrations of 50% (more adequate for bearings design), it was obtained a final porosity of 26.36%. It was observed the right choice of milling time helps getting average pore diameter values with narrow standard-deviation, with good definition of Normal Curve indicating a homogeneous condition (Figure 4). SEM images of material samples sections show the better distribution (50%) and pores size, considering different concentrations of the sucrose. It was observed a uniform distribution in the structure with average values porous diameter between 0.04 and 30 µm as presented in Figure 5.

The experimental set-up for the permeability measurements (permeameter) was based on the pressure drop based on the norm ISO 4022:1987[15] considering the Darcy and Forchheimer Laws applied to porous medium as showed in Figure 7. The equipment has a cylindrical chamber manufactured with a Polyvinyl Chloride tube with 1 m length. The air flow in the tube is perpendicular to the porous material sample (Figure 3) that is positioned inside a holder.

ISSN 2179-3956

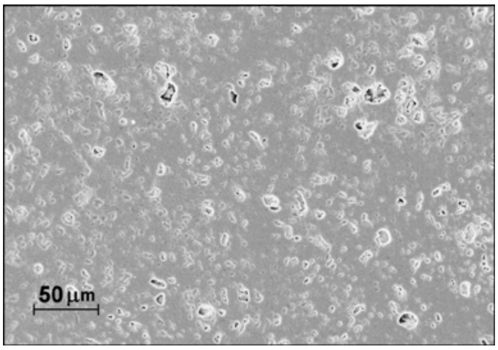


(a) Normal Curve (50% Sucrose).

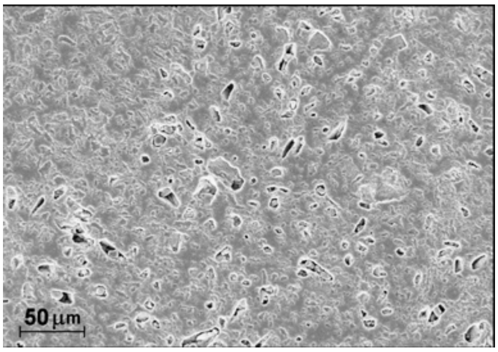


(b) Normal Curve (40% Sucrose).

Figure 5 - Mercury porosimetry results for the sucrose concentrations.



50% Sucrose (1000x).



40% Sucrose (1000x).

Figure 6 – Sections of material samples (SEM images).

The holder is located in the middle distance between the pressure gauges that are fixed along the chamber, allowing measurements independently of the sample thickness. The long feeding tube connected to the sample holder leads the fluid directly to the sample in order to reduce possible flow turbulence. Two air filters are located in the compressed air line, one for pre-filtering (40 µm grid) and another for coalescence with 0.01 µm grid. During the experimental tests, the room remained in ambient temperature, with temperature around 25°C, the air density was assumed to be 1.079 kg/m^3 and air dynamic viscosity to be 1.83 x 10^{-5} Pa.s.

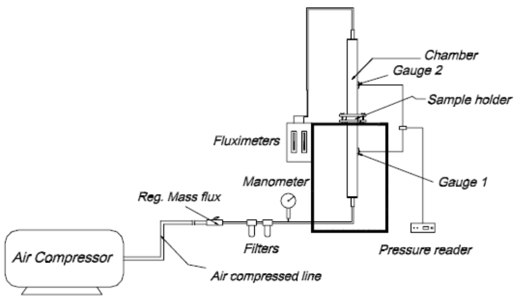


Figure 7 – (a) Schematic view of the experimental set-ups for measuring material permeability.

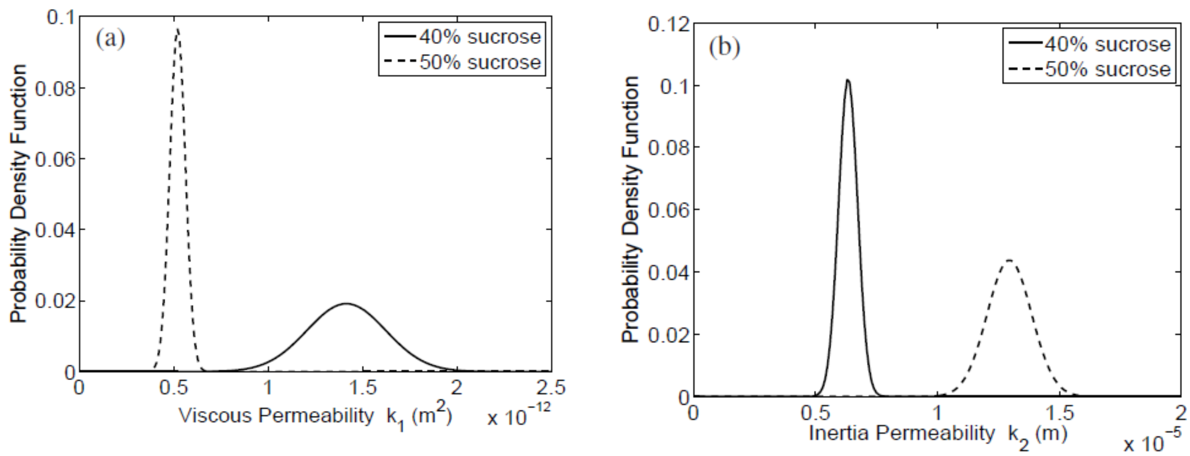
ISSN 2179-3956

The ceramic material samples are composed of alumina with porous forming agent, which in this case is sucrose. In Silveira, Purquerio and Fortulan,(16) several samples of different concentrations of pore-forming agents were manufacturing. Following these samples were analyzed by mercury porosimetry and SEM images. The permeability measurements indicate the sucrose concentrations of 40% and 50% more suitable for bearing design. So that, the porous forming agent concentrations of 40% and 50% sucrose are tested. After testing four material samples of each sucrose concentration, one obtained the averaged values of permeability shown in Table 1.

Table 1 – Statistical parameters of the permeability coefficients obtained from tested samples

	material	viscous permeability $k_1 \times 10^{-12}$ (m^2)	inertia permeability $k_2 \times 10^{-6}$ (m)
mean value	40% sucrose	1.4150	0.5213
	50% sucrose	6.3375	12.950
standard deviation	40% sucrose	0.2087	0.0413
	50% sucrose	0.3903	0.9110

A normal distribution approximation of the experimental values resulted in the curves is presented in Figure 8. As one can see in Figure 8 (a) the values of viscous permeability k_1 presents a smaller dispersion for the material with 50% sucrose in comparison to that of the 40% sucrose material. The mean value of k_1 for the 50% sucrose material is also lower than that of the 40% sucrose material. On the other hand, the inertia permeability coefficient k_2 presents an opposite behavior Figure 8 (b). Lower mean value, as well as lower dispersion, is presented by the 40% sucrose material in comparison to the values of k_2 for the 50% sucrose material.



(a) **(b)**

Figure 8 – Probability density functions of the permeability coefficients of the tested sample: (a) viscous permeability; (b) inertia permeability.

4 NUMERICAL RESULTS AND DISCUSSION

Considering the experimental results of material permeability distribution, one will investigate the effects of variation of the material permeability coefficients on the resultant bearing non dimensional load capacity. Given operational conditions and

ISSN 2179-3956

values for k_1 and k_2 the aerostatic pressure distribution is obtained from Equation (1) and bearing load capacity is estimated by integrating the pressure distribution over the bearing sliding surface. The variability of the permeability coefficients is considered by applying a Monte Carlo simulation based on the statistical data of the samples. Hence, for a given operational condition of the bearing, the values of parameters k_1 and k_2 are normally distributed according to the statistical data obtained experimentally, and for each set of parameters the Reynolds equation is solved and bearing load capacity is calculated. In this work, one adopted a universe of 3000 samples in the Monte Carlo simulation.

Considering nominal values, the bearing load capacity depends on the bearing non dimensional parameters Γ and Φ as illustrated in Figure 9 (a). As one can see, there is a linear region below $\Phi = 10$ where Forchheimer assumption is not relevant (quadratic pressure-flow assumption), and only Darcy assumption can be considered. In the region $1 < \Phi < 10$, there is a transition between linear and nonlinear regions, where weak nonlinearity is perceived. For $\Phi < 1$, nonlinearity becomes important. Considering these results, one simulated three different operational conditions of the bearing: linear ($\Gamma = 55$, $\Phi = 100$), threshold ($\Gamma = 60$, $\Phi = 4$), and nonlinear ($\Gamma = 400$, $\Phi = 0.01$). Γ values were chosen aiming at having maximum nominal load capacity for a given Φ value.

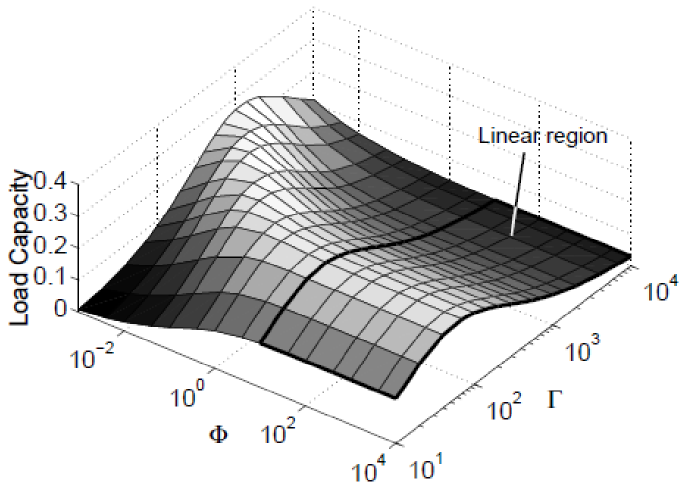
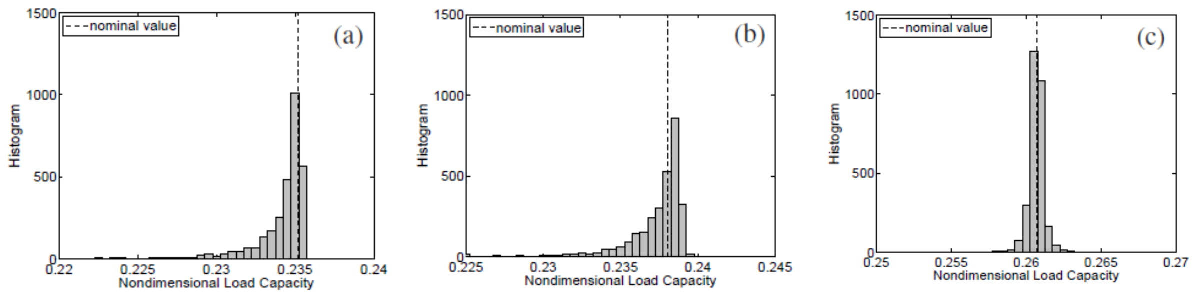


Figure 9 – Non dimensional load capacity as a function of bearing parameters Γ and Φ.[1]

The results of Monte Carlo simulation in the bearing load capacity estimation are shown in Figures 10 and 11 for the two tested materials. As one can see, the bearing composed of porous material with 40% sucrose presents a higher dispersion in load capacity than that composed of porous material with 50% sucrose, in all simulated cases.

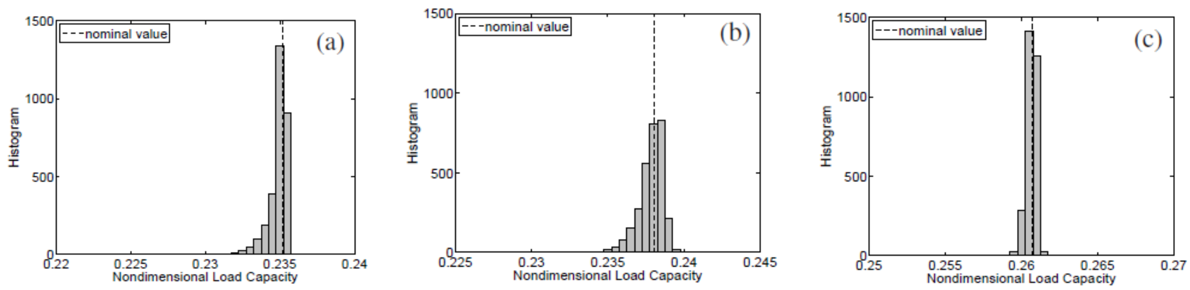
ISSN 2179-3956



(a) **(b)** **(c)**

Figure 10 – Bearing load capacity historgram for the porous material with 50% sucrose: (a) linear condition; (b) threshod condition; (c) non linear condition.

Analyzing the operational conditions, one can see that the threshold region is that of higher dispersion of results Figure 10 (b) and Figure 11 (b) where there is more probability to have lower bearing load capacity. On the other hand, if one designs the bearing in the nonlinear region, one will expect the lowest dispersion of load capacity results Figure 10 (c) and Figure 11(c). However, bearing load capacity dispersion remained below 2 % in all cases (materials and conditions).



(a) **(b)** **(c)**

Figure 11 – Bearing load capacity historgram for the porous material with 40% sucrose: (a) linear condition; (b) threshod condition; (c) non linear condition.

This is an important observation, considering bearing design, because it reveals that bearing load capacity is not significantly influenced by material property variations. Therefore, one can expect that the aerostatic porous bearing present load capacity close to that it was designed for.

5 CONCLUSION

The design of aerostatic porous bearings, and the definition of the bearing static characteristics, depends on a set of operating conditions, geometric parameters, and material properties. Considering porous bearing, material permeability coefficients are the parameters that will affect more importantly the fluid flow behavior in the bearing, and consequently, the resultant bearing characteristics. In this work, a Monte Carlo simulation was adopted in order to investigate the effects of permeability variation on the resultant bearing load capacity. As a result, one observed that this characteristic is not strongly affected, thus helping a more robust design of the bearing. From the obtained results, bearing designers can expect less than 2% of variation in the bearing load capacity due to variation in material permeability.

Acknowledgements

The authors gratefully acknowledge the FAPESP and CNPq for the financial support to this work and the Empresa Brasileira de Pesquisa Agropecuária (EMBRAPA – Instrumentação Agropecuária) and Assistant Professor Carlos A. Fortulan of EESC-USP for the support given to this project.

References

1 NICOLETTI, R., SILVEIRA, Z.C., PURQUERIO, B.M. Modified Reyonlds equation for aerostatic porous radial bearings with quadratic Forchheimer pressure-flow assumption, *Journal of Tribology* – ASME (2008). v. 130, n. 3, p. 031701-1/12. 2008.
2 HANNON, W.M.; BRAUN, M.J.; HARIHARAN, S.I. Universal Reynolds equation for variable properties fluid-film lubrication and variable geometry self-acting bearings, *Tribology Transactions* - STLE (2004), v. 47, n. 2, p. 171-181. 2004.
3 CHANG, L., ZHAO, Y. On the sensitivity of the asperity pressures and temperatures to the fluid pressure distribution in mixed-film lubrication. *Journal of Tribology*- ASME (2008), v. 122, n. 1, p. 77-85. 2000.
4 CHUN, S.M. Thermohydrodynamic lubrication analysis of high-speed journal bearing considering variable density and variable specific heat, *Tribology International - Elsevier (2004)*, v. 37, n. 5, p. 405-413. 2004.
5 FILLON, M., DMOCHOWSKI, M.W., DADOUCHE, A., Numerical study of the sensitivity of tilting-pad journal bearing performance characteristics to manufacturing tolerances: steady-state analysis, *Tribology Transactions* - STLE (2007), v. 50, No. 3, p. 387-400. 2007.
6 OGRODNIK, J., OODWIN, M.J., BANCROFT, G. A., XU, W. The stability analysis of hydrodynamic journal bearings allowing for manufacturing tolerances, *Proceedings of the 2009 IEEE International Conference on Measuring Technology and Mechatronics Automation,* IEEE (2009), Zhangjiajie, China, April 11-12, p. 168-171. 2009.
7 RUBINSTEIN, R.Y. *Simulation and the Monte Carlo method*. John Wiley & Sons, New York, 1981.
8 WANG, J. VLAHOPOULOS, N., MOURELATOS, Z. P, EBRAT, O. VAIDYANATHAN, K. Probabilistic and sensitivity analyses for the performance characteristics of the main bearings in an operating engine due to variability in bearing properties, *International Journal of Vehicle Design,* v. 40, n. 4, Interscience, p. 265-298. 2004.
9 CUSANO, C. The effect of variable permeability on the performance characteristics of porous bearings, Wear, v. 23, no. 1, p. 55-62. Elsevier 1973.
10 BUJURKE, N.M., PATIL, H. P. The effect of variable permeability and rotating on the performance characteristics of porous bearings, *Wear*, v. 155, n. 1, p. 7-14. Elsevier 1992.
11 BUJURKE, N.M., PATIL, H. P. The effects of variable permeability and roughness of porous bearings. *International Journal of Mechanical Sciences*, v. 34, n. 5, Elsevier (1992), p. 355-362. 1992.
12 MANN, R., ANDROUTSOPOULOS, G. P., GOLSHAN, H. Application of a stochastic network pore model to oil-bearing rock with observations relevant to oil recovery, *Chemical Engineering Science*, v. 36, n. 2, Elsevier (1981), p. 337-346. 1981.
13 PETROPOULOS, J. H., PETROU, J. K., KANELLOPOULOS, N. K. Explicit relation between relative permeability and structural parameters in stochastic pore networks, *Chemical Engineering Science*, v. 44, n. 12, Elsevier (1989), p. 2967-2977. 1989.

ISSN 2179-3956

14 D'AGOSTINO, V., RUGGIERO, A., SENATORE, A. Unsteady oil film forces in porous bearings: analysis of permeability effect on the rotor linear stability, *Meccanica*, v. 44, n. 2, Springer (2009), p. 207-214. 2009.
15 ISO 4022:1987 (E): Permeable sintered metal materials – Determination of fluid permeability. *International Organization for Standardization.* 9p. 1987.
16 SILVEIRA, Z.; PURQUERIO, B.M.; FORTULAN, C.A. Projeto, Fabricação e Caracterização de Estruturas de Cerâmicas Porosas para Aplicação em Mancais Aerostáticos. *In: Anais do 17th Congresso Brasileiro de Ciência e Engenharia de Materiais* – CBECiMat 2006, Foz do Iguaçu, PR, Brasil. 2006.