THE DEVELOPMENT AND USE OF THE LARSEN-MILLER EQUATION AND HOLLOMAN-JAFFE PARAMETERS FOR STRESS-RELIEVING OPERATIONS: A REVIEW¹

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Abstract

The results of the stress relieving process are dependent on the temperature and time which are correlated through Holloman's (Holloman-Jaffe) parameter which is a measure of the thermal effect of the process and processes with the same Holloman's parameter exhibit the same effect. Another similar commonly used expression used in evaluating the stress relief of spring steels is the Larson-Miller equation. Although the Larsen-Miller Equation and the Holloman Jaffe parameter are known, they are seldom cited in many heat treating texts. Furthermore, their origin and limitations of use are even less well-known. A review of the metallurgical, origin, use and limitations of these expressions is provided here.

Key words: Heat treatment; Tempering; Stress relief; Lanson-Miller equation.

ASPECTOS DO DESENVOLVIMENTO E USO DA EQUAÇÃO DE LARSEN-MILLER E PARÂMETROS DE HOLLOMAN-JAFFE NO PROCESSO DE ALÍVIO DE TENSÕES

Resumo

Os resultados do processo de alívio de tensões são dependentes da temperatura e do tempo que são correlacionados pelos parâmetros de Holloman (Holloman-Jaffe), que é a medida do efeito térmico do processo. Processos com o mesmo parâmetro, exibem o mesmo efeito. Outra expressão comumente usada para avaliar o alívio de tensões em aços mola é a equação de Larsen –Miller. Embora tanto os parâmetros quanto a equação sejam conhecidos, são citados ao acaso em muitos textos de tratamentos térmicos. Além disso, suas origens e limitações de uso são bem menos conhecidas. Neste artigo será fornecida uma revisão metalúrgica, além da origem e discussão das limitações de uso dessas expressões.

Palavras-chave: Tratamento térmico; Revenimento; Alívio de tensões; Equação de Larsen-Miller

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INTRODUCTION

The tempering process is dependent on both temperature and time at temperature. Improper selection of these process parameters can potentially affect temper embrittlement, non-optimal stress relief, stress corrosion cracking, hardness, impact toughness, transformation of retained austenite and residual stress. Furthurmore, appropriate temperature and time selection is dependent on carbon content and steel alloy chemistry. Figure 1 illustrates the interdependence of tempering time and temperature as a function of carbon content. (1,2) The so-called tempering parameter (P) correlates the time-temperature interrelationship and is also referred to as the Larsen-Miller parameter or the Holloman-Jaffe parameter:

$$P = T(C + \log t) \times 10^{-3}$$

where: t is the time (hours) at temperature T (in Kelvin) and 18 is the value for C recommended by Grange and Baughman for all carbon and low alloy steels and this is the form most commonly encountered for this equation currently. ⁽⁶⁾

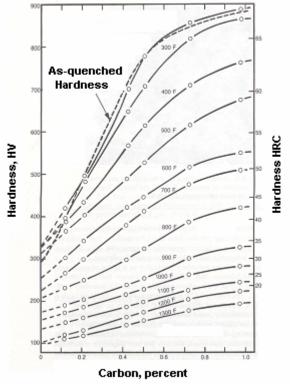


Figure 1. Interrelationship of carbon content and hardness after tempering at various temperatures for one hour.

However, tempering times are also dependent on steel alloy chemistry. The correlation of tempering time and temperature dependency for carbon and alloy steels, where total alloy content was less than 5% and carbon was present from 0.2-1.0%, was developed by Holloman and Jaffe⁽⁵⁾ and Grange and Baughman⁽⁶⁾ for tempering times of 343-649°C. Figure 2 illustrates the variation of hardness with the carbon content of the steel and on P (which is dependent on tempering temperature and time.⁽²⁾ Figure 3 illustrates the tempering parameter variation with tempering time and temperature.⁽²⁾

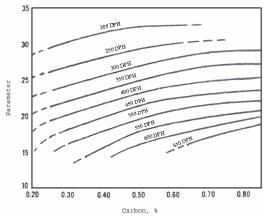




Figure 2. Variation of hardness with the carbon content of carbon steel

Figure 3. Tempering parameter nomogram for C=18

The Holloman-Jaffe approach to calcluate a tempering nomogram (t vs hardness) for a low alloy steel involves the construction of a plot of P for a carbon steel (with the total carbon content of interest) from: $t = T(18 + log t) \times 10^{-3}$ vs hardness using values from Figure 2. To calculate the hardness for the tempering parameters of: 20, 22, 24, 26, 28 and 30, one multiplies the factor shown in Table 1 for the alloy of interest. For example, consider AISI 4340 with the following composition: 0.42% C, 0.78% Mn, 0.24% Si, 1.85% Ni, 0.81% Cr, 0.27% Mo. In this case, adjust the tempering parameter for AISI 1042 using the factors from Table 1 for P = 20, then: DPH = $(5 \times 1.85) + (50 \times 0.81) + (20 \times 0.27) = 55$. This would be repeated for other P values and a hardnes vs P curve can be constructed. To determine the hardness of AISI 4340 after tempering for 5 hours at 538°C (1000°F), the P value would be 27.3 yielding a hardness of 37.5 using the nomogram constructed for 4340 steel (not shown).

If the tempering temperature is raised to 650°C (1200°F), one can determine the time required to give the same hardness (0.7 hours) from:

$$T_1(18 + \log t_1) = T_2(18 + \log t_2)$$

Clearly, these are invaluable computational resources which have been used for a number of years and are often encountered in the literature. However, the actual historical development and metallurgical basis for these relationships and their limitations are much less well known. The objective of this paper is to provide a review of this information as it relates to steel heat treatment.

Table 1. Factors for hardness prediction of tempered martensite

Element	Range	Factor at Indicated Value of Parameter					
	(%)	20	22	24	26	28	30
Mn	0.85-2.1	35	25	30	30	30	25
Si	0.3-2.2	65	60	30	30	30	30
Ni	≤ 4	5	3	6	8	8	6
Cr	≤ 1.2	50	55	55	55	55	55
Мо	≤ 0.35	40	90	160	220	240	210
		20 ²	45 ²	80 ²	110 ²	120 ²	105 ²
V ¹	≤ 0.2	0	30	85	150	210	150

^{1.} For AISi-SAE Cr-V steels, may not apply when V is the only carbide former present.

^{2.} If 0.5-1.2 Cr is present, also use this factor

Note boron factor is 0

DISCUSSION

In 1945, Holloman and Jaffe studied the effect of tempering temperature and time on the as-tempered hardness of $six^{(6)}$ carbon steels with varying carbon contents ranging from $0.31-1.51\%.^{(5)}$ They assumed that hardness was "some" function of the diffusion equation:

$$H=f(te^{-Q/RT})$$

where: H is hardness, t is the tempering time, T is the tempering temperature (absolute), R is the ideal gas constant, Q is a constant characteristic of the steel and f is a proportionality constant. This equation was used to fit their data for tempering temperature and time. However, it was found that the value of Q was also dependent on hardness and:

$$t_o = te^{-Q/RT}$$

where t_o is a constant dependent on the steel. In logarithmic form, this equation becomes:

Q = RT (In t – In
$$t_0$$
) = f_2 H
H= f_3 ($e^{RTIn t/to}$)

$$H = f(T \log t/t_0) = f(T (\log t - \log t_0)) = fT(C + \log t)$$

For a constant hardness and assuming that temperature is constant during tempering:

$$T_1 (C + \log t_1) = T_2 (C + \log t_2)$$

Solving for the material constant C:

$$-C = \frac{T_1 \log t_1 - T_2 \log t_2}{T_1 - T_2}$$

$$\frac{T_1}{T_2} = \frac{C + \log t_1}{C + \log t_2}$$

$$C = -log t_0$$

The equation: H = T (C + log t) is known as the Holloman-Jaffe equation. Using this equation to fit their hardness and tempering time/temperature correlations, they concluded:⁽⁵⁾

- The data fit the equation to ± 1 Rc hardness units except when graphitization occurred irregardless of the initial microstructure.
- The calue of C varied somewhat for different steels and decreased linearly with the carbon content of a steel grade.
- The value of C was not critical in correlating the interdependence of tempering temperature and time.
- Holloman and Jaffe proposed that C = 19.5 for carbon and alloy steels with carbon contents of 0.25-0.4%; C = 15 for tool steels with carbon contents of 0.9-1.2%.
- If there is no secondary hardening, Rockwell and Brinell hardness varies almost linearly with C over a "considerable range".

Nehrenberg developed tempering curves for a series of stainless steels and AISI 4340 (see Figure 4).⁽⁷⁾ Based on these data and earlier work of Holloman and Jaffe among others, Nehrenberg developed a tempering parameter (P) which permitted the prediction of as-tempered hardness as a function of tempering temperature and time. It was reported that these data best fit the equation:

$$P = T (20 + log t)$$

where T is the absolute temperature and t is the tempering time in hours.

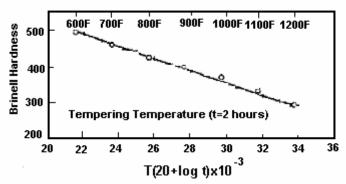


Figure 4. Nehrenberg master tempering curve for AISI 4340 which was derived from a 0ne (1) inch (25.4 mm) cross-section size of a steel bar that was tempered after oil quenching from 1500°F (816°C).

Larsen and Miller examined the effect of tempering time and temperature on creep rupture data for a range of steels that included a low-carbon steel, stainless steel, an alloy steel and also a non-ferrous, titanium alloy. They also developed their equation using the value of C = 20 reported earlier by Nehrenberg. In hindsight, perhaps the so-called Larsen-Miller equation should have been called the Nehrenberg equation. However, Nehrenberg assumed that the equation was stress dependent and whereas Larsen and Miller assumed that that relationship was independent of stress and, in fact, reported that it was independent of the material.

One of the most common uses of the Larsen-Miller equation is not for determining tempering process parameters but for analysis of creep data. When the Larsen-Miller equation is used for creep, the value for T is assummed to be the testing temperature (absolute) and the value for t is time for rupture (failure). The creep rupture rate (r) which inversely proportional to time (t):

$$r \propto 1/t$$

then:

$$\frac{\Delta H}{R}$$
 = T (C+ log t)

where ΔH is the activation energy, R is the ideal gas constant, t is the time to rupture, and C = 20 as originally reported by Larsen and Miller. Although the value of C = 20 is often assumed, the actual value for C at a given stress (σ) can be calculated at two test temperatures (T_1 and T_2) and the log t for each value of T from Eq. 4 above Eq 7 using the value of the Larsen-Miller parameter calculated from: P = T (C + Log t) as illustrated in Figure 5. (In the case of creep calculations, the property being calculated is creep stress, not hardness.)

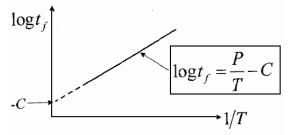


Figure 5 Graphical determination of the material parameter C from creep rupture data.

Eriksson analyzed the effect of heating and cooling on the used of the Larsen-Miller equation (also referred to as the Holloman-Jaffe equation in that paper) was addressed. (8) Based on this work, a modification of the Larsen-Miller equation was

reported for the calculation of additional equivalent time at the tempering temperature for the heating and cooling portions of the overall tempering cycle:

$$T/2.3 k (20 - lok k)$$

where: T is the tempering temperature and k is the heating or cooling rate in K/hr. In a recent literature debate between Furillo, et al.⁽⁹⁾ and DiMelfi.⁽¹⁰⁾ Furillo, et. Al. argued that the Larsen-Miller parameter was dependent on both temperature and stress and that the value of C, as determined from creep data, may vary between 8 to 57 depending on the steel alloy and thus should not always be assummed to be 20. Conversely, DiMelfi argued that the Larsen-Miller equation, as it related to utility for creep predictions, was a function of stress only but that conclusion was contested by Furillo, et.al. From this debate, it is evident that whatever the correct position, when the Larsen-Miller equation is used for creep calculations, care should be taken in the routine selection of C=20, although it would seem to be adequate for tempering process parameter predictions.

Grange and Baughman built upon the Holloman-Jaffe approach. (6) Applying the calues for C reported by Holloman and Jaffe to the much more extensive data available at the U.S. Laboratory database, they found that the variations of C was not non-critical but in fact could vary greatly, depending on the steel alloy with the greatest variations (up to a factor of 4) observed for alloy steel grades. Therefore, an average value of C under these conditions had little meaning. (6) The significance of varying the valuer of C is evident in Figure 2. Grange and Baughman did report however, that the best fit of the equation through their data was obtained with a C value of 18 and that the correlation was improved using DPH diamond pyramid hardness), now known as HV or Vickers hardness, values. This variation was reportedly due to the greater penetration depth of measurement used to obtain Rockwell C hardness. Therefore, DPH was used as the hardness measurement for their work.

Figure 6 shows that using C = 18 provides a generally good fit for a range of carbon steels (AISI 1030, 1050 and 1080). However, as the hardness decreases, there is an appant tendency for the curves to merge which is probably because of the formation of few large carbides as the hardness approaches that of ferrite.

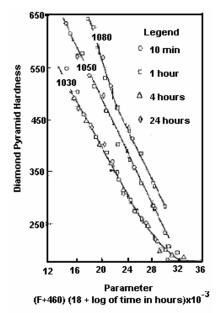


Figure 6. Tempering curves for AISI 1030, 1050 and 1080 carbon steels.

Alloying elements affect the tempering behavior of steel and different alloying elements affect the tempering behavior diferently. Figure 7 and Figure 8 are tempering curves for AISI 4340 and 5140 steels respectively. To more clearly observe the net effect of the alloying element, the tempering curve for the corresponding carbon steel with the same carbon content is also shown on each figure. By determining the softening effect of addition of different alloying elements to carbon steel by comparing the tempering curve and the corresponding carbon steel curve, an "alloy factor" (expressed as DPH increments) can be calculated. A summary of these values is provided in Table 1. These factors can be used in addition to the tempering curves for carbon steel shown in Figure 3 to construct tempering curves for an alloy of interest. The following comments relative to these values were provided by Grange and Baughman: (6)

- Alloying elements exhibit their effect at different tempering temperatures when compared to other element.
- Since the effect of an alloying element is not directly proportional to the amount that may be present, relatively high concentrations may indicate an excessively high hardness using the alloy factors in Table 1. Therefore, a concentration range for which the use of the given alloy factor is valid was provided.
- When two or more alloying elements are present, these elements tend to compete with each other as carbide formers and therefore the hardness calculated using the alloy factors may be greater than actually observed. This effect was addressed for Mo by decreasing its alloying factor by ½.
- At tempering temperatures lower than 343°C (650°F) (Tempering Parameter values less than 20), the resulting hardness of tempered martensite is influenced by rate of cooling during the prior quenching process and therefore by workpiece cross-section size and shape in addition to quench severity. This is due to the relative amount of retained austenite, particularly at lower carbon concentrations. This can also be affected by slower cooling through the martensite transformation range (M_s-M_f). The relative amount of "quench tempering" increases as the (M_s-M_f) temperature range increases. Therefore, due to potential unreliability, alloy factors below Tempering Parameter values of 20 are not provided.

Alloy factors are not included for for Tempering Parameter values of 32 (and higher) because this corresponds to a tempering process of one (1) hour at 704° C (1300°F) which is greater than the Ac_1 temperature for nickel-containing steel. Although the Holloman-Jaffe equation can be used with reasonable success to estimate tempering time and temperature processes, the use of a single equation can lead to significant errors for most alloy steels. The approach provided by Grange and Baughman by the use of alloy factors, was shown to work well for a wide range of carbon and alloy steels if its use is restricted to 343-649°C (650-1200°F) and a limited range of alloy concentrations.

There are various reports describing the use of the Larsen-Miller (Holloman-Jaffe) equations. Gingras and Grenier have developed a successful Excel-based graphical program to estimate hardness afforded by different forced air tempering ovens, alloy, load size, etc.⁽¹¹⁾

Recently, the successful use of a modified Holloman-Jaffe equation has been reported for induction tempering which has the form: (12,13)

$$P = T (14.44 + log t)$$

where T is absolute temperature in Rankine (°F + 460) and t is the induction tempering time in seconds. (Holloman and Jaffe reported earlier that if one adds 3.57 to the C value for tempering time in seconds, one obtains the C-value for tempering time in hours. (5) In this case, the equation would become P = T (18 + log t) which is the equation recommended by Grange and Baughman. (6)

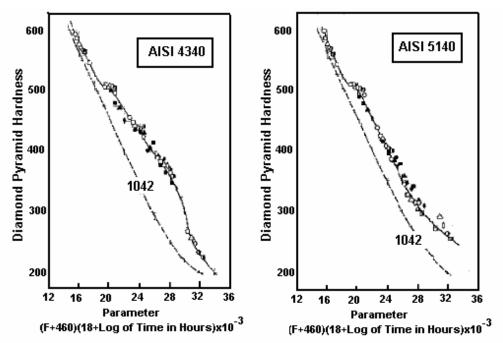


Figure 7. Grange and Baughman curve for AISI 4340 steel.

Figure 8. Grange and Baughman curve for AISI 5140 steel.

Jarl, et. al. used the Holloman-Jaffee equation to determine equivalent tempering cycles in a molten lead bath relative to induction tempering by estimating tensile strength for a spring wire carbon steel $(0.56\% \ C)$. They found that although the variation in tensile stress was within \pm 20 Mpa compared to experimental variation of \pm 25 Mpa, the predicted tensile stress was consistently lower than the experimental value.

Zikeev, et.al. studied the hydrogen resistance of a Russian steel (18Kh1G1MF = 0.19% C, 0.84% Mn, 0.29% Si, 1.27% Cr, 0.28% Mo, 0.06% V, 0.016% Al, 0.017% P, and 0.005% S).⁽¹⁴⁾ The equation for the Holloman-Jaffe parameter that was used for their work was:

where: P is the Holloman-Jaffee parameter, T is the tempering temperature (K), t is the tempering time (hr), and a is a constant = 40. Their work showed that they could obtain a linear correlation of tensile strength with the tempering parameter (P). In addition, they found that when the Holloman-Jaffee parameter was >38, the susceptability to hydrogen embrittlement for this steel increased dramatically as shown in Figure 9.⁽¹⁴⁾

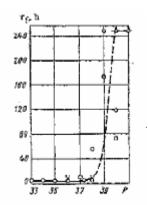


Figure 9. Correlation of hydrogen embrittlement of 18Kh1G1MF steel to the Holloman-Jaffe parameter.

The Larsen-Miller equation has been used to estimate the effect of shorter times at higher temperatures on the tensile strength and ductility dual phase steel wire used for fastener production. The results of this work showed that by increasing the aging temperature resulted in an increase in tensile strength. Thus, aging at 250°C for 1000 hr was equivalent to 210°C for 100,000 hours. In addition there was a corresponding increase in ductility as illustrated in Figure 10. (16)

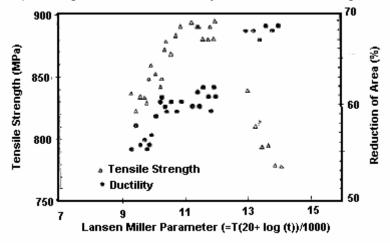
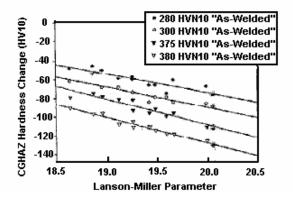


Figure 10. Effect of tensile strength and ductility variation on a dual phase spring wire after aging at an elevated temperature

Another example of the use of the Larsen-Miller equation is to estimate the change in hardness of the CGHAZ (coarse-grained heat affected zones) of V-microalloyed steel. This study showed that post-weld heat treatment at 635°C with holding times between 3 and 15 hours resulted in approvimately parallel lines when the change in hardness between the initial hardness and as-tempered hardness versus the Larsen-Miller parameter (Figure 11) which indicates the tempering effect is independent of the initial hardness of the CGHAZ. Figure 12 shows illustrates the correlation of as-tempered hardness of the CGHAZ as a function of the Larsen-Miller parameter. Ramirez, et. al. suggested that since the Larsen-Miller equation was developed as an indicator of a single reaction, the agreement of these welding data with the Larsen-Miller parameter suggests that the tempering process of the CGHAZ is also controlled by a single metallurgical reaction.



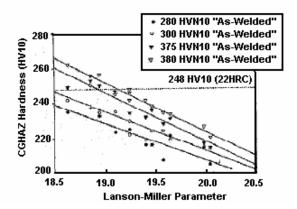


Figure 11. ffect of tempering on the change of CGHAZ hardness as function of as-welded hardness and theLarsen-Miller parameter.

Figure 12- Hardness of the CGHAZ as function of the as-welded hardness and the Lanson-Miller parameter.

CONCLUSIONS

An overview of the development and metallurgical basis of the Larsen-Miller and Holloman-Jaffe equations was provided here in addition to various selected examples of its use. These equations Historically, the Holloman-Jaffe equation was reported first and it was based on relatively limited steel tempering data. Also, there was not a single Holloman-Jaffe tempering parameter but instead a limited number of parameters were recommended which varied with the type of steel.

The initial work by Holloman and Jaffe was followed by Nehrenberg who first reported the single equation known as the Larsen-Miller equation today. This equation was developed to aid in tempering of stainless steel. Nehrenberg examined the correlation of his tempering data and found that the C-value of 20 best fit his tempering data. Larsen and Miller follwed the work of Nehrenberg and they used the Nehrenberg equation to predict creep rupture of various steels. This equation has traditionally been known as the Larsen-Miller equation and has been used to successfully predict stress relief conditions as well as tempering process parameters for many steels.

Holloman and Jaffe's work was followed by Grange and Baughman of U.S. Steel who developed the the process of using graphical representation of the Holloman-Jaffe equation for a wide range of steels. In their work, they reported that a C-value of 18 best fit carbon steel data. Alloy effects on the tempering process may then be estimated by using a table of alloy factors derived from actual tempoering data. This approach was reported to work well for a very broad range of alloy steels. However, the Grange and Baughman approach is seldom utilized today.

The terms Larsen-Miller and Holloman-Jaffe equations are often used synonymously today although technically they are not the same even if they are used for predicting tempering process parameters.

Finally, a number of examples illustrating the successful use of both the Holloman-Jaffe and Larsen-Miller equations to address various metallurgical problems.

REFERENCES

- G. Krauss, "Chapter 8 Tempering of Steel", in <u>Principles of Heat Treatment of Steel</u>, 1980, American Society for Metals, Metals Park, OH, p. 187-227.
- 2 R.F. Kern and M.E. Suess, "Chapter 6 Hardenability and Tempering Parameter", in <u>Steel Selection: A Guide for Improving Performance and Profits</u>, 1979, J.Wiley and Sons, New York, NY, p. 82-113.
- F.R. Larsen and J. Miller, "A Time-Temperature Relationship for Rupture and Creep Stresses", Trans. For the American Society of Mechanical Engineers, 1952, Vol. 74, p. 765-775.
- 4 A.K. Sinha, "Chapter 14 Tempering", in <u>Physical Metallurgy Handbook, 2003</u>, McGraw-Hill, New York, NY, p. 14.45-14.50.
- 5 J.H. Hollomon and J.H. Jaffe, "Time-Temperatures Relations in Tempering Steel", Transactions of the American Institute of Miningand Metallurgical Engineers, 1945, Vol. 162, p. 223-249.
- 6 R.A. Grange and R.W. Baughman, "Hardness of Tempered Martensite in Carbon and Low Alloy Steels", in Transactions of American Society for Metals, 1956, Vol. XLVIII, p. 165-197.
- 7 A.E. Nehrenberg, "Master Curves Simplify Stainless Tempering", Steel, 1950, Vol. 127, October 23, p. 72-76.
- 8 T.F. Gulvin, D. Scott, D.M. Haddrill, and J. Glen, "Paper No. 621 The Influence of Stress Relief on the Properties of C and C-Mn Pressure-Vessel Plate Steels", J. of West of Scotland Iron and Steel Institute, 1972-1973, Vol. 80, p. 149-175.
- 9 F.T. Furillo, S. Purushothaman, and J.K. Tien, "Further Discussion on Understanding the Larsen-Miller Parameter", Scripta Metallurgica, 1978, Vol. 12, p. 331-332.
- 10 R.J. DiMelfi, "Comments on Understanding the Larsen-Miller Parameter by F.T. Furillo, S. Purushothaman, and J.K. Tien", Scripta Metallurgica, 1978, Vol. 12, p. 327-329.
- 11 11 R. Gingras and M. Grenier, "Software Assists in Optimizing Tempering Process", Industrial Heating, 2005, Vol. 72, No. 12, December, p. 49-52.
- 12 Anon. "Techcommentary Induction Heat Treatment", Newsletter published by EPRI Center for Materials Fabrication, 1985, Vol. 2, No. 2, Available from EMF, 505 King Ave., Columbus, OH 43201, USA.
- 13 Anon. "Techcommentary Induction Tempering", Newsletter published by EPRI Center for Materials Fabrication, 1985, Vol. 2, No. 4, Available from EMF, 505 King Ave., Columbus, OH 43201, USA
- 14 V.N. Zikeev, Yu.V. Kornyushchenkova, and V.V. Izvil'skii, "Influence of the Temperature-Time Parameter of Tempering on the Properties of 18Kh1G1MF Steel Resistant to Hydrpgen Embrittlement", Metal Science and Heat Treatment, 1984, Vol. 26, No. 1-2, p. 99-102.
- M. Jarl, T. Berntsson, E. Sapcanin, and B. Azizi, "Replacement of Lead Baths for the Tempering of Spring Wire", Scandinavian J. of Metallurgy, 2003, Vol. 32, p. 241-246.
- 16 A.J. Trowsdale and S.B. Pritchard, "Dual Phase Steel High Strength Fasteners without Heat Treatment", Internet: http://www.corusautomotive.com/file_source/StaticFiles/Microsites/Automotive/Technical/Final_CBM_Dupla_Paper.pdf
- 17 J.E. Ramierez, S. Mishael, and R. Shockley, "Properties and Sulfide Stress Cracking Resistance of Coarse-Grained Heat-Affected Zones in V-Microalloyed X60 Steel Pipe", Welding Journal, 2005, July, p. 113-s 123-s.